Name ____ Test 3, Fall 2019

1) A linear transformation is given by the rule $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 3x_2 \\ x_3 \end{bmatrix}$. Find the kernel of T. (10 points)

2) Determine whether or not the linear transformation from the previous problem is onto and justify your answer. (10 points)

3) Suppose a linear transformation satisfies the equations below. Find $[T]$. (10 points)

$$T\left(\begin{bmatrix}2\\0\end{bmatrix}\right)=\begin{bmatrix}2\\6\end{bmatrix} \quad T\left(\begin{bmatrix}0\\3\end{bmatrix}\right)=\begin{bmatrix}6\\12\end{bmatrix}$$

4) Let $B_1=\left\{\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}0\\3\end{bmatrix}\right\}$ and $B_2=\left\{\begin{bmatrix}5\\4\end{bmatrix},\begin{bmatrix}0\\6\end{bmatrix}\right\}$. Find a formula for $[I]_{B_1}^{B_2}$, the change of basis matrix from B_1 to B_2. (10 points)

5) Let B_1 and B_2 both be bases for $\mathbb{R}^3$. Let T be a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^3$ with matrix representation A under the standard basis. Find a formula for $[A^{-1}]_{B_1}^{B_2}$. (10 points)

6) Find a diagonalization of the matrix below. (10 points)

$$\begin{bmatrix} 7 & -3 \\ 6 & -2 \end{bmatrix}$$

7) Below a basis is given. Find an orthogonal version of this basis. (10 points)

$$\left\{ \begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \right\}$$

8) Suppose that a 8×8 matrix is invertible. What is something else that can be said about the corresponding system of equations $A\vec{x} = \vec{0}$? (5 points)

9) Suppose that a 8×8 matrix is invertible. What is something else that can be said about the corresponding linear transformation T? (5 points)

10) Suppose that a 8×8 matrix is invertible. What is something else that can be said about the matrix itself? (5 points)

11) Suppose that a linear transformation $T: \mathbb{R}^4 \to \mathbb{R}^4$ is onto. What is the maximum dimension of the solution set to $A\vec{x} = \vec{0}$, where A is the matrix representation of T. (5 points)

12) Suppose that a linear transformation $T: \mathbb{R}^5 \to \mathbb{R}^6$ is one-to-one. What is the maximum dimension of the solution set to $A\vec{x} = \vec{0}$, where A is the matrix representation of T. (5 points)

13) Suppose that a linear transformation $T: \mathbb{R}^7 \to \mathbb{R}^7$ is invertible. What is the minimum rank of A, where A is the matrix representation of T. (5 points)