Name ____ Quiz 3

1) Suppose A is a 5×5 matrix and $\vec{v}$ & $\vec{w}$ distinct 5×1 vectors. If both $\vec{v}$ and $\vec{w}$ are solutions to $A\vec{x} = \vec{0}$, how is the largest possible value for $\text{rank}(A)$?

The fact that we have two solutions, tell us that $A\vec{x} = \vec{0}$ has at least 1 free variable. Hence the maximum possible rank is 4.

In $\mathbb{R}^3$, define the two bases below.

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix} \right\}; \; B_2 = \left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$$

2) Given the vector $[\vec{x}]_{B_1} = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$, find $[\vec{x}]_S$

$$\begin{bmatrix} 1 & 7 & 0 \\ 2 & 8 & 3 \\ 3 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}_{B_1} = \begin{bmatrix} 23 \\ 40 \\ 28 \end{bmatrix}_S$$

3) Given the vector $[\vec{x}]_{B_2} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$, find $[\vec{x}]_{B_1}$

$$\begin{bmatrix} 1 & 7 & 0 \\ 2 & 8 & 3 \\ 3 & 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 1 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}_{B_1} = \begin{bmatrix} -\frac{487}{33} \\ \frac{46}{33} \\ \frac{334}{33} \end{bmatrix}_{B_2}$$