Name ____ Test 1, Spring 2019

1) Multiply the matrices below, or state why it is impossible.
(+1/-3 points)

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & 3 \\ 0 & 5 \end{bmatrix}$$

2) Given the system of equations below, write the corresponding matrix equation $A\vec{x} = \vec{b}$.
(+2/-3 points)

$$2x_1 + 9x_2 - 4x_3 = -10$$
$$-x_1 - 3x_2 + 5x_3 = 5$$

3) Assume B and D are 3×2 matrices, while C is a 2×3 matrix. Simplify the expression below or explain why it is not possible.

(+2/-3 points)

$$B(C^T + D)$$

4) Compute $\vec{u} - \vec{v}$, given the definitions below.

(+1/-5 points)

$$\vec{u} = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}; \vec{v} = \begin{bmatrix} -4 \\ 1 \\ 5 \end{bmatrix}$$

5) Given the definitions below, express $\vec{b}$ as a linear combination of $\vec{a}_1$ and $\vec{a}_2$, or explain why it is not possible.

(+3/-1 points)

$$\vec{a}_1 = \begin{bmatrix} 4 \\ -6 \end{bmatrix}; \vec{a}_2 = \begin{bmatrix} -6 \\ 9 \end{bmatrix}; \vec{b} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

6) Row reduce the matrix below.

(+2/-2 points)

$$\begin{bmatrix} -2 & -4 & 0 \\ 6 & 12 & 7 \end{bmatrix}$$

7) Solve the system of equations below.

(Hint: look at the previous question)

(+2/-3 points)

$$\begin{aligned} -2x_1 - 4x_2 &= 0 \\ 6x_1 + 12x_2 &= 7 \end{aligned}$$

8) Are the vectors $\begin{bmatrix} -2 \\ 6 \end{bmatrix}$, $\begin{bmatrix} -4 \\ 12 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 7 \end{bmatrix}$ linearly independent? Justify your answer.

(+3/-2 points)

True or False: circle the correct answer. A statement is true if it is *always* true; false if it is *ever* false.

T or F 9) The set $\mathbb{R}^{m\times n}$ together with the usual definition of matrix addition and scalar multiplication is a vector space.

T or F 10) Four distinct vectors in $\mathbb{R}^3$ will span $\mathbb{R}^3$.

T or F 11) Three distinct nonzero vectors in $\mathbb{R}^4$ will be linearly independent.

T or F 12) The set of vectors $\left\{\begin{bmatrix} a \\ b \end{bmatrix} : a, b \in \mathbb{Q}\right\}$ is a subspace of $\mathbb{R}^2$.

(+1/-1 point each)

13) A square matrix B is called <u>idempotent</u> if $B^2 = B$. Find three examples of 2×2 matrices that are idempotent.

(+3/-1 points)

14) Find three vectors that are in the span of $\begin{bmatrix} 2 \\ 6 \end{bmatrix}$ and $\begin{bmatrix} 9 \\ 15 \end{bmatrix}$.

(+3/-2 points)

15) Find a vector that is not in the span of $\begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

(+3/-2 points)

For the problems on this page, use the matrix below on the left. It's row reduced echelon form is shown on the right.

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ -2 & -3 & -1 & 4 \\ 1 & 4 & -2 & 4 \\ 2 & 2 & 2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

16) Find the row space of A, $RS(A)$.

(+2/-3 points)

17) Find the column space of A, $CS(A)$.

(+3/-2 points)

18) Find the null space of $A, NS(A)$.

(+4/-2 points)

For the problems on this page, use the matrix below on the left. Its row reduced echelon form is shown on the right.

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ -2 & -3 & -1 & 4 \\ 1 & 4 & -2 & 4 \\ 2 & 2 & 2 & -4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

19) Determine if the homogeneous system $A\vec{x} = \vec{0}$ has any nontrivial solutions.

(+4/-3 points)

20) Find a basis for the span of the columns of A.

(+2/-2 points)

21) Expand the set $\left\{\begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}\right\}$ to a basis of $\mathbb{R}^3$

(+2/-3 points)

22) Let A be an 7×5 matrix. You know that $A\vec{x} = \vec{b}$ has no solutions. What else can you say?

(Maximum +4/-4 points) (You may list as many statements as you like, each insightful statement is worth +2 or -2 points.)