Name ____ Test 2, Spring 2019

Non-technology portion

1) Find a formula for $[\vec{x}]_S$, given the information below.
(+2/-3 points)

$$B = \left\{ \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}, [\vec{x}]_B = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 1 \\ 3 & 0 & 1 \\ 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}_B$$

2) Find a formula for $\vec{v}$, given the information below.
(+2/-3 points)

$$G = \left\{ \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \end{bmatrix} \right\}, [v]_G = \left\{ \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix}_G$$

3) Find the determinant of the matrix below.

(+2/-5 points)

$$\begin{bmatrix} 1 & -5 \\ 0 & -4 \end{bmatrix}$$

$$\begin{vmatrix} 1 & -5 \\ 0 & -4 \end{vmatrix} = (1)(-4) - (-5)(0) = -4$$

4) Find the determinant of matrix A, given the information below.

(+4/-1 points)

$$A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} B, \qquad |B| = 2$$

$$|A| = \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} \cdot |B| = 1 \cdot 2 = 2$$

5) Given that $T(\vec{x}) = A\vec{x}$ and the matrix A below. Is T one-to-one? Justify your answer.
(+3/-2 points)

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 4 \end{bmatrix}$$

T is not one to one because the matrix representation of T has columns without pivots. These correspond to free variables in the equation $A\vec{x} = \vec{b}$, which yields multiple solutions to the same $T(\vec{x}) = \vec{b}$ equation.

6) Given that $T(\vec{x}) = A\vec{x}$ and the matrix A below. Is T onto? Justify your answer.
(+4/-2 points)

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 4 \end{bmatrix}$$

T is onto because the matrix representation of T has a pivot in each row. These correspond to being able to solve each of the equations in $A\vec{x} = \vec{b}$, which yield a solution to every $T(\vec{x}) = \vec{b}$ equation.

7) Find the rank of the matrix below.
(+3/-5 points)

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 4 \end{bmatrix}$$

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8) An 19×16 matrix has a null space of dimension 3. What is the rank of A?
(+3/-3 points)

In row reduced form, the matrix will have 3 columns without pivots. $16 - 3 = 13$, so the rank of the matrix is 13.

9) Find the determinant of the matrix below.

(+4/-2 points)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 4 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

The determinant is -6

This can be found by applying the elementary row operation that interchanges rows 3 and 4. Doing so multiplies the determinant by -1. The resulting matrix obviously has determinant 6.

10) Given that $T(\vec{x}) = A\vec{x}$ and the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & 2 \end{bmatrix}$ below, find $T\left(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}\right)$.

(+1/-5 points)

$$T\left(\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

11) Prove that the function $f(x) = 3x + 2$ is one-to-one.
(+4/-1 points)

Assume $f(x_1) = f(x_2)$
$\therefore 3x_1 + 2 = 3x_2 + 2$
$\therefore 3x_1 = 3x_2$
$\therefore x_1 = x_2$

12) Let A be an 7×7 matrix. You know that $A\vec{x} = \vec{0}$ has only one solution. What else can you say? (Maximum +8/-8 points) (You may list as many statements as you like, each insightful statement is worth +2 or -2 points.)

This is the big theorem. Here is a selection of statements that we can say. Note that you only want to say the true ones, or rephrase the false ones into their negation.

Statements on solutions and free variables:

The equation $A\vec{x} = \vec{b}$ has a free variable – FALSE

The equation $A\vec{x} = \vec{b}$ has a unique solution for each $\vec{b}$ - TRUE

The equation $A\vec{x} = \vec{0}$ has infinitely many solutions – FALSE

The equation $A\vec{x} = \vec{0}$ has only $\vec{x} = \vec{0}$ as a solution - TRUE

Statements on linear dependence and independence of rows and columns:

The rows of A are linearly dependent – FALSE

The rows of A are linearly independent – TRUE

The columns of A are linearly independent – TRUE

The columns of A are linearly dependent – FALSE

Statements on the matrix itself

In echelon form, every row has a pivot – TRUE

In echelon form every column has a pivot – TRUE

In echelon form A has a row of zeroes – FALSE

The row space spans $\mathbb{R}^7$ – TRUE

The column space spans $\mathbb{R}^7$ – TRUE

The null space is nontrivial – FALSE

The matrix is invertible – TRUE

The matrix is a product of elementary matrices – TRUE

The matrix is singular – FALSE

The rank of the matrix is 7 – TRUE

The determinant of the matrix is zero – FALSE

Statements on the corresponding linear transformation

The corresponding linear transformation is one-to-one – TRUE

The corresponding linear transformation is onto – TRUE

Technology portion

13) Find the inverse of the matrix below.
(+5/-5 points)

$$\begin{bmatrix} 1 & 0 & 8 & 4 \\ 0 & 2 & 0 & 0 \\ 1 & 5 & 3 & 4 \\ 0 & 7 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 8 & 4 \\ 0 & 2 & 0 & 0 \\ 1 & 5 & 3 & 4 \\ 0 & 7 & 0 & 1 \end{bmatrix}^{-1} = \frac{1}{10}\begin{bmatrix} -6 & 100 & 16 & -40 \\ 0 & 5 & 0 & 0 \\ 2 & 5 & -2 & 0 \\ 0 & -35 & 0 & 10 \end{bmatrix}$$

14) Find $[\vec{x}]_{B_2}$, given the information below.
(+5/-5 points)

$$B_1 = \left\{\begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 7 \end{bmatrix}\right\}, B_2 = \left\{\begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}\right\}, [\vec{x}]_{B_1} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 1 & 1 \\ 3 & 5 & 0 \\ 4 & 1 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 2 & 2 & 2 \\ 6 & 0 & 1 \\ 4 & 1 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}_{B_1} = \begin{bmatrix} -\frac{56}{27} \\ \frac{31}{9} \\ \frac{619}{27} \end{bmatrix}_{B_2}$$