Name ____ Test 1, Fall 2020

1) Multiply the two matrices below or state why they cannot be multiplied. (15 points)

$$\begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & -1 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2+8 & 3-2 \\ 6-4 & 9+1+4 \end{bmatrix} = \begin{bmatrix} 10 & 1 \\ 2 & 14 \end{bmatrix}$$

2) Find the null space of the matrix below. (16 points)

$$\begin{bmatrix} 1 & 0 & 3 & 0 & -2 \\ 0 & 2 & 4 & 0 & 8 \\ 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 + 3x_3 - 2x_5 = 0$
$x_1 = -3x_3 + 2x_5$

$2x_2 + 4x_3 + 8x_5 = 0$
$x_2 = -2x_3 - 4x_5$

$x_4 - 2x_5 = 0$
$x_4 = 2x_5$

The Null space is:

$$\left\{ \begin{bmatrix} -3x_3 + 2x_5 \\ -2x_3 - 4x_5 \\ x_3 \\ 2x_5 \\ x_5 \end{bmatrix} : x_3, x_5 \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 2 \\ -4 \\ 0 \\ 2 \\ 1 \end{bmatrix} x_5 : x_3, x_5 \in \mathbb{R} \right\} = span\left(\left\{ \begin{bmatrix} -3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 0 \\ 2 \\ 1 \end{bmatrix} \right\} \right)$$

3) Reduce the matrix below to reduced row echelon form. (16 points)

$$\begin{bmatrix} 2 & 4 & 6 & 8 & 24 \\ 1 & 2 & 3 & 4 & 13 \\ 0 & 1 & 2 & 1 & 6 \\ 0 & 1 & 2 & 2 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & 6 & 8 & 24 \\ 1 & 2 & 3 & 4 & 13 \\ 0 & 1 & 2 & 1 & 6 \\ 0 & 1 & 2 & 2 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 3 & 4 & 12 \\ 1 & 2 & 3 & 4 & 13 \\ 0 & 1 & 2 & 1 & 6 \\ 0 & 1 & 2 & 2 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 3 & 4 & 12 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 & 6 \\ 0 & 1 & 2 & 2 & 6 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 3 & 4 & 12 \\ 0 & 1 & 2 & 2 & 6 \\ 0 & 1 & 2 & 1 & 6 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 \to \frac{1}{2}R_1$ $\qquad$ $R_2 \to R_2 - R_1$ $\qquad$ $R_2 \leftrightarrow R_4$

$$\sim_R \begin{bmatrix} 1 & 2 & 3 & 4 & 12 \\ 0 & 1 & 2 & 2 & 6 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 6 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 2 & 6 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 6 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$ $\qquad$ $R_1 \to R_1 - 2R_2$ $\qquad$ $R_3 \to -R_3$ $\qquad$ $R_2 \to R_2 - 2R_3$

$$\sim_R \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 6R_4$

4) Answer the questions below (3 points each)

(A) Let A be a 3×3 matrix that is a product of elementary matrices. How many solutions does $A\vec{x} = \vec{0}$ have?

1

(B) If A is a 5×4 matrix and $\vec{b}$ a nonzero vector such that $A\vec{x} = \vec{b}$ has infinitely many solutions, what is the minimum possible number of zero rows A has after it is row-reduced?

2

(C) Let A be a 2×2 invertible matrix. How many solutions does $A\vec{x} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ have?

1

(D) Let A be a 3×3 invertible matrix and B a 3×3 singular matrix. What is the dimension of the row space of the augmented matrix $[A|B]$?

3

(E) Let A be a 5×5 matrix and assume that $A\vec{x} = \vec{0}$ and $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 0 \\ 0 \end{bmatrix}$ have infinitely many solutions, but $A\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$ has no solutions. What is the minimum and the maximum number of pivots A can have when in row reduced echelon form?

At minimum there is 1, at maximum there are 4.

5) Let $\vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ -5 \\ 0 \end{bmatrix}$. Find $5\vec{v}$. (8 points)

$$5\begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ -5 \\ 0 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 15 \\ 20 \\ -25 \\ 0 \end{bmatrix}$$

6) Are $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 2 & 3 & 2 \end{bmatrix}$ and $\begin{bmatrix} -8 & 5 & 2 \\ 6 & -4 & -1 \\ -1 & 1 & 0 \end{bmatrix}$ inverses of each other? Either explain your answer or show your work. (8 points)

They are inverses of each other:

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 2 & 3 & 2 \end{bmatrix}\begin{bmatrix} -8 & 5 & 2 \\ 6 & -4 & -1 \\ -1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For the problems on this page, you may be interested in the fact that

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 8 & 7 \\ 2 & 4 & 6 & 8 & 5 \\ 1 & 2 & 4 & 5 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 & 3 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

7) Find the row space of the matrix below. Avoid using redundant vectors when possible. (7 points)

$$\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 8 & 7 \\ 2 & 4 & 6 & 8 & 5 \\ 1 & 2 & 4 & 5 & 0 \end{bmatrix}$$

$$span(\{[1 \quad 0 \quad 0 \quad 3 \quad 0], [0 \quad 1 \quad 0 \quad -1 \quad 0], [0 \quad 0 \quad 1 \quad 1 \quad 0], [0 \quad 0 \quad 0 \quad 0 \quad 1]\})$$

$$= span\left(\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 3 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}^T, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}^T\right\}\right)$$

8) Find the dimension of the vector space below. (8 points)

$$span\left(\left\{\begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \\ 6 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 8 \\ 8 \\ 5 \end{bmatrix}\right\}\right)$$

3

9) Given $A = \begin{bmatrix} 1 & 7 \\ 2 & 3 \end{bmatrix}$ And $f(x) = 3x + 2$, find $P(A)$. (7 points)

$$3\begin{bmatrix} 1 & 7 \\ 2 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 21 \\ 6 & 11 \end{bmatrix}$$