Name ____________________________________________________________ Test 2, Fall 2020

1) Find the determinant of the matrix below. (15 points)

$$\begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 5 & 8 & 1 & 0 & 0 \\ 6 & 6 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 & 1 \\ 0 & 2 & 2 & 3 & 4 \end{bmatrix}$$

2) Given the two bases below, find the change of basis matrix that converts information from coordinate vectors in $B_2$ to coordinate vectors in $B_1$, denoted by $[I]_{B_2}^{B_1}$. You do not need to perform the arithmetic. (10 points)

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\} \quad B_2 = \left\{ \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \right\}$$

3) Given the two bases and vector below, find a formula for $\vec{x}_{B_2}$. You do not need to perform the arithmetic. (10 points)

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\} \quad B_2 = \left\{ \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \right\} \quad \vec{x}_{B_1} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}_{B_1}$$

4) Answer the questions below (3 points each)

(A) Let $A$ be a $3 \times 3$ matrix such that $A\vec{x} = \vec{0}$ has one free variable. What is $|A|$?

(B) Let $A$ be a $3 \times 5$ matrix such that, when row reduced, has only 1 pivot. What is the dimension of the null space of $A$, often written $\dim\big(NS(A)\big)$?

(C) Let $A$ be a $5 \times 3$ matrix and $T$ be the corresponding linear transformation. Assume $T$ is one-to-one. How many pivots does $A$ have, when row reduced?

(D) Let $A\vec{x} = \vec{0}$ be a system of equations that has multiple solutions. How many solutions does it have?

(E) Let $A$ be a $11 \times 7$ matrix. There are 6 linearly independent rows. What is the rank of $A$?

5) Given the linear transformation below, determine whether or not it is *one-to-one* and justify your answer. (10 points. 3 for the answer; 7 for the reasoning)

$$T: \mathbb{R}^4 \to \mathbb{R}^4$$
$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

6) Given the linear transformation below, determine whether or not it is *onto* and justify your answer. (10 points. 3 for the answer; 7 for the reasoning)

$$T: \mathbb{R}^4 \to \mathbb{R}^4$$
$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

7) Find the determinant of the matrix below (5 points)

$$\begin{bmatrix} 6 & 5 \\ 4 & 2 \end{bmatrix}$$

8) Find the product below (10 points)

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 4 \\ 4 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 & 3 \\ 5 & 4 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

9) Given the system of equations below. Determine how many solutions it has. (5 points)

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

10) Given the system of equations below. Express its solution set in proper notation. (10 points)

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$