Name ____ Test 2, Fall 2020

1) Find the determinant of the matrix below. (15 points)

$$\begin{bmatrix} 0 & 2 & 0 & 0 & 0 \\ 5 & 8 & 1 & 0 & 0 \\ 6 & 6 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 & 1 \\ 0 & 2 & 2 & 3 & 4 \end{bmatrix}$$

$$\begin{vmatrix} 0 & 2 & 0 & 0 & 0 \\ 5 & 8 & 1 & 0 & 0 \\ 6 & 6 & 0 & 1 & 0 \\ 0 & 4 & 0 & 0 & 1 \\ 0 & 2 & 2 & 3 & 4 \end{vmatrix} = -2 \begin{vmatrix} 5 & 1 & 0 & 0 \\ 6 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 2 & 3 & 4 \end{vmatrix} = (-2)(-1) \begin{vmatrix} 5 & 1 & 0 \\ 6 & 0 & 1 \\ 0 & 2 & 3 \end{vmatrix}$$

$$= (-2)(-1)\left(5 \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} - 1 \begin{vmatrix} 6 & 1 \\ 0 & 3 \end{vmatrix}\right) = (-2)(-1)\big(5(-2) - 1(18)\big)$$

$$= (-2)(-1)(-28) = -56$$

2) Given the two bases below, find the change of basis matrix that converts information from coordinate vectors in B_2 to coordinate vectors in B_1, denoted by $[I]_{B_2}^{B_1}$. You do not need to perform the arithmetic. (10 points)

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\} \quad B_2 = \left\{ \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \right\}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 2 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 7 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

3) Given the two bases and vector below, find a formula for $\vec{x}_{B_2}$. You do not need to perform the arithmetic. (10 points)

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} \right\} \quad B_2 = \left\{ \begin{bmatrix} 7 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} \right\} \quad \vec{x}_{B_1} = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}_{B_1}$$

$$\begin{bmatrix} 7 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}_{B_1}$$

4) Answer the questions below (3 points each)

(A) Let A be a 3×3 matrix such that $A\vec{x} = \vec{0}$ has one free variable. What is $|A|$?

0

(B) Let A be a 3×5 matrix such that, when row reduced, has only 1 pivot. What is the dimension of the null space of A, often written $\dim(NS(A))$?

4

(C) Let A be a 5×3 matrix and T be the corresponding linear transformation. Assume T is one-to-one. How many pivots does A have, when row reduced?

3

(D) Let $A\vec{x} = \vec{0}$ be a system of equations that has multiple solutions. How many solutions does it have?

∞

(E) Let A be a 11×7 matrix. There are 6 linearly independent rows. What is the rank of A?

6

5) Given the linear transformation below, determine whether or not it is *one-to-one* and justify your answer. (10 points. 3 for the answer; 7 for the reasoning)

$$T: \mathbb{R}^4 \to \mathbb{R}^4$$
$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

It is not one to one because there is a column without pivots.

6) Given the linear transformation below, determine whether or not it is *onto* and justify your answer. (10 points. 3 for the answer; 7 for the reasoning)

$$T: \mathbb{R}^4 \to \mathbb{R}^4$$
$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}\right) = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

It is not onto because there is a row without pivots.

7) Find the determinant of the matrix below (5 points)

$$\begin{bmatrix} 6 & 5 \\ 4 & 2 \end{bmatrix}$$

$$6 \cdot 2 - 4 \cdot 5 = 12 - 20 = -8$$

8) Find the product below (10 points)

$$\begin{bmatrix} 1 & 4 & 5 \\ 2 & 3 & 4 \\ 4 & -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 & 3 \\ 5 & 4 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 30 & 18 & 8 \\ 23 & 16 & 10 \\ -1 & 4 & 14 \end{bmatrix}$$

9) Given the system of equations below. Determine how many solutions it has. (5 points)

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

∞

10) Given the system of equations below. Express its solution set in proper notation. (10 points)

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 0 \end{bmatrix}$$

$x_3 = 2 - 2x_4$
$x_1 = 4 - 3x_2$

x_2 and x_4 are free.

The solution set is:

$$\left\{ \begin{bmatrix} 4 - 3x_2 \\ x_2 \\ 2 - 2x_4 \\ x_4 \end{bmatrix} : x_2, x_4 \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} 4 \\ 0 \\ 2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ -2 \\ 4 \end{bmatrix} : x_2, x_4 \in \mathbb{R} \right\}$$