Name ____ Test 3, Fall 2020

Before you start, note that the last page contains some formulas that might help you on some problems. You can tear that page off for reference as you complete the test if you like.

1) Find the eigenvalues and eigenspaces of the matrix below. Please label or circle your answers. (15 points)

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & 0 \\ -2 & 0 & -2 \end{bmatrix}$$

2) Let $B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \end{bmatrix} \right\}$ and $B_2 = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \end{bmatrix} \right\}$. Define the linear transformation $T: \mathbb{R}^2_S \to \mathbb{R}^2_S$ as below.

$$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_S \right) = \begin{bmatrix} x_1 + x_2 \\ 7x_2 \end{bmatrix}_S$$

Find a formula for $[T]_{B_1}^{B_2}$, the matrix representation of T in the given bases. (15 points)

3) Answer the questions below. (3 points each)

(A) Let A be a 6×6 matrix with eigenvalues $0, 2, 2, 4, 5,$ and 6. How many solutions does $A\vec{x} = \vec{0}$ have?

(B) Let A be a 4×6 matrix whose corresponding linear transformation is onto. What is the rank of A?

(C) Let A be a 5×4 matrix. And let $\vec{b}_1$ and $\vec{b}_2$ both be nonzero vectors such that $A\vec{x} = \vec{b}_1$ has a unique solution, but $A\vec{x} = \vec{b}_2$ has no solutions. What can be said about the dimension of the row space of A?

(D) Let A be a 5×4 matrix. And let $\vec{b}_1$ and $\vec{b}_2$ both be nonzero vectors such that $A\vec{x} = \vec{b}_1$ has multiple solutions, but $A\vec{x} = \vec{b}_2$ has no solutions. What can be said about the dimension of the row space of A?

(E) If $T: \mathbb{R}^5 \rightarrow \mathbb{R}^{17}$ is a linear transformation that is one-to-one and A is the matrix that represents T, what is the dimension of the null space of A?

4) Find a formula for $\begin{bmatrix} 1 & 4 & -2 \\ 0 & -1 & 0 \\ -2 & 0 & -2 \end{bmatrix}^{10}$ that requires at most 3 matrix multiplications. It may involve other arithmetic or matrix operations. (10 points)

5) Let T_1 and T_2 be defined as below. Find a formula for $[T_2 \circ T_1]$. (5 points)

$T_1 : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $[T_1] = \begin{bmatrix} 1 & 8 \\ 2 & 7 \end{bmatrix}$.

$T_2 : \mathbb{R}^2 \to \mathbb{R}^2$ is given by $[T_2] = \begin{bmatrix} 5 & 4 \\ 10 & 8 \end{bmatrix}$.

6) Let T be defined as below. Find the kernel of T. (10 points)

$T_1 : \mathbb{R}^4 \to \mathbb{R}^3$ is given by $[T] = \begin{bmatrix} 1 & 2 & 4 & 2 \\ 0 & 4 & 8 & 4 \\ 1 & 4 & 8 & 4 \end{bmatrix}$.

7) Multiply the two matrices below. (10 points)

$$\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 1 & -1 \end{bmatrix}$$

8) Row reduce the matrix below. (5 points)

$$\begin{bmatrix} 2 & 8 & 4 \\ 4 & 10 & 8 \end{bmatrix}$$

9) Answer yes (Y) or no (N) to the following questions. Is it possible to multiply these matrices?.
(2 points each)

Y N (A) $\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \begin{bmatrix} 2 & 8 & 4 \\ 4 & 10 & 8 \end{bmatrix}$

Y N (B) $\begin{bmatrix} 2 & 8 & 4 \\ 4 & 10 & 8 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$

Y N (C) The matrix on the left is 5×6; the matrix on the right is 6×7

10) Is $\begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}$ a linear combination of $\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$? If so, is it unique? (9 points)

The following formulas might or might not be useful on some problems on this test.

$$\begin{bmatrix} 1 & 1 \\ 0 & 7 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 8 \\ 2 & 7 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 \\ 10 & 8 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0.8 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & 0 \\ -2 & 0 & -2 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & -2 \\ 0 & -1 & 0 \\ -2 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -2 \\ 0 & 3 & 0 \\ 2 & 4 & 1 \end{bmatrix} \begin{bmatrix} -3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 & -2 \\ 0 & 3 & 0 \\ 2 & 4 & 1 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 4 & 6 & 4 \\ 5 & 7 & 9 & 7 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 & 2 \\ 0 & 4 & 8 & 4 \\ 1 & 4 & 8 & 4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$