1) Multiply the matrices below. (10 points)

$$\begin{bmatrix} 1 & 2 & 4 \\ -2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 5 \\ 3 & -1 & 3 \\ -1 & 2 & 1 \end{bmatrix}$$

2) Find the null space of the matrix below. (10 points)

$$\begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3) Row reduce the matrix below. (10 points)

$$\begin{bmatrix} 1 & 2 & 4 \\ -2 & -4 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

4) Consider a 4×6 matrix A that has 3 pivots. (2 points each)

  (A) How many solutions does $A\vec{x} = \vec{0}$ have?

  (B) How many free variables does the equation $A\vec{x} = \vec{0}$ have?

  (C) If $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 7 \end{bmatrix}$ has no solutions, how many solutions does it have?

  (D) If $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 7 \end{bmatrix}$ has a solution, how many solutions does it have?

  (E) Does A have an inverse? True or false.

5) Find the inverse of the matrix below. (10 points)

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 0 & 1 & 3 \end{bmatrix}$$

6) Find the product below. (5 points)

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 5 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

7) Determine whether or not the vectors below are orthogonal. Justify your answer. (5 points)

$$\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

8) Below is a matrix equation. Write down the corresponding system of homogeneous equations. (5 points)

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 7 \end{bmatrix}$$

9) Use the formula $\|\vec{v}\| \cdot \|\vec{w}\| \cdot \cos(\theta) = \vec{v} \bullet \vec{w}$ to find the angle between the two vectors below. You do not need to simplify your answer. (5 points)

$$\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix}$$

10) Given the two vectors below, find $2\vec{v} - 3\vec{w}$. (5 points)

$$\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \vec{w} = \begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix}$$

11) Find $\vec{v}^T\vec{w}$, given the two vectors below. (5 points)

$$\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \vec{w} = \begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix}$$

12) Given the information below, solve $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ (5 points)

$$A = \begin{bmatrix} 4 & 5 & -4 \\ 2 & 4 & -3 \\ -1 & -1 & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 4 \\ 2 & -1 & 6 \end{bmatrix}$$
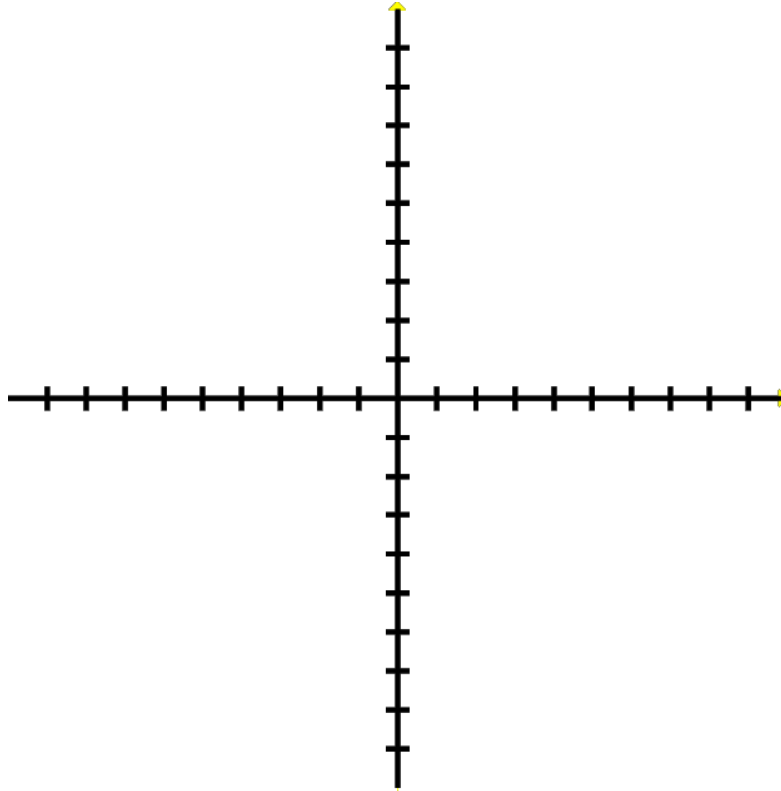
13) Find the length of the vector below. (5 points)

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 5 \end{bmatrix}$$

14) Graphically illustrate the solution to the system of equations below. (5 points)

$$2x + y = 4$$
$$3x - y = 5$$

15) Find the transpose of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 0 & 2 & 8 \\ 3 & 4 & 5 & 0 \\ 1 & 0 & 2 & 1 \end{bmatrix}$$