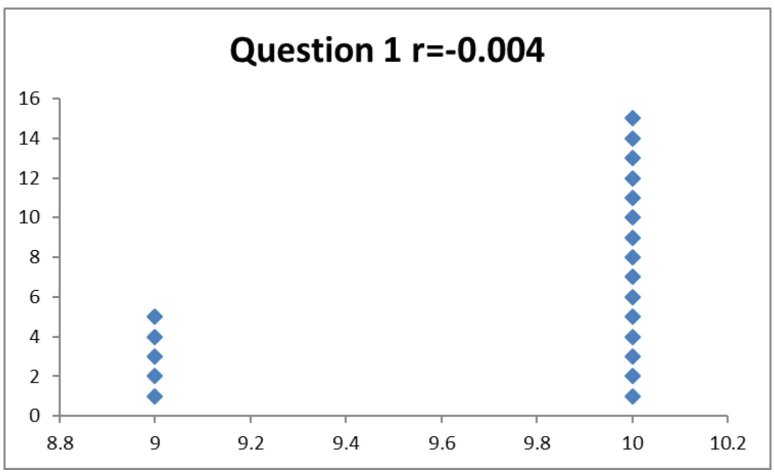
Name ____ Test 1, Spring 2020

1) Multiply the matrices below. (10 points)

$$\begin{bmatrix} 1 & 2 & 4 \\ -2 & 1 & 3 \\ 1 & 2 & 1 \end{bmatrix}\begin{bmatrix} 4 & 3 & 5 \\ 3 & -1 & 3 \\ -1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 9 & 15 \\ -8 & -1 & -4 \\ 9 & 3 & 12 \end{bmatrix}$$



2) Find the null space of the matrix below. (10 points)

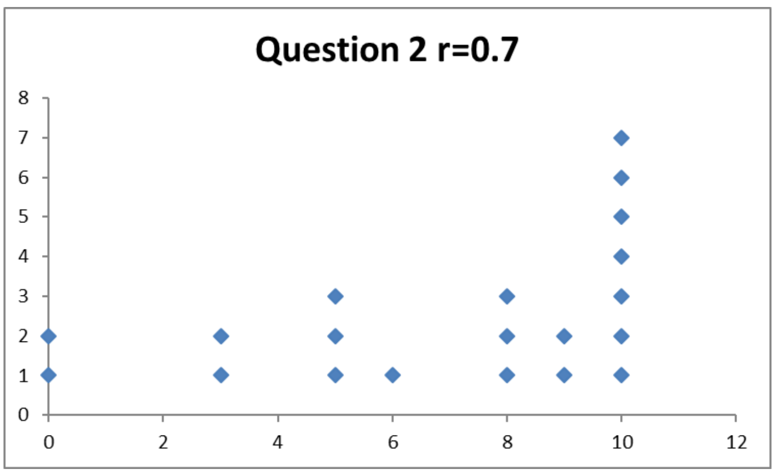
$$\begin{bmatrix} 1 & 2 & 0 & -2 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

x_2 and x_4 are free. The two equations from the homogeneous system of equations give us:

$$x_1 + 2x_2 - 2x_4 = 0$$
$$x_3 + 4x_4 = 0$$

So the solution set, which is the null space of the matrix is:

$$\left\{ \begin{bmatrix} -2x_2 + 2x_4 \\ x_2 \\ -4x_4 \\ x_4 \end{bmatrix} : x_2, x_4 \in \mathbb{R} \right\} = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 2 \\ 0 \\ -4 \\ 1 \end{bmatrix} x_4 : x_2, x_4 \in \mathbb{R} \right\}$$



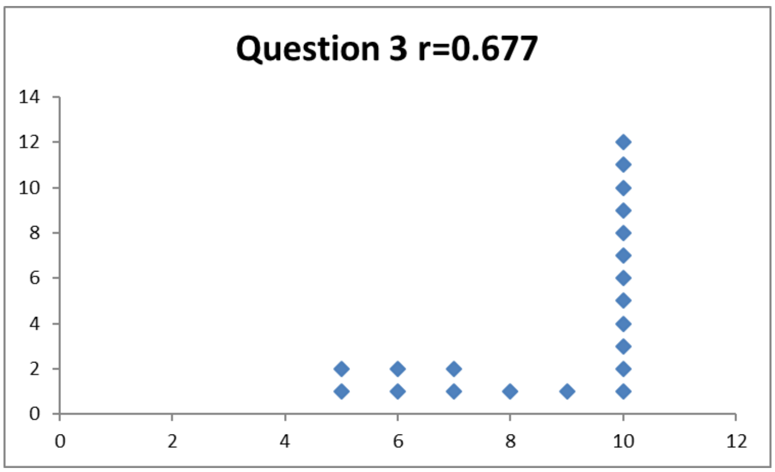
3) Row reduce the matrix below. (10 points)

$$\begin{bmatrix} 1 & 2 & 4 \\ -2 & -4 & 3 \\ 1 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ -2 & -4 & 3 \\ 1 & 2 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 11 \\ 0 & 0 & -3 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 4 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R_2 \to R_2 + 2R_1 \quad R_2 \to \frac{1}{11}R_2 \quad R_1 \to R_1 - 4R_2$$

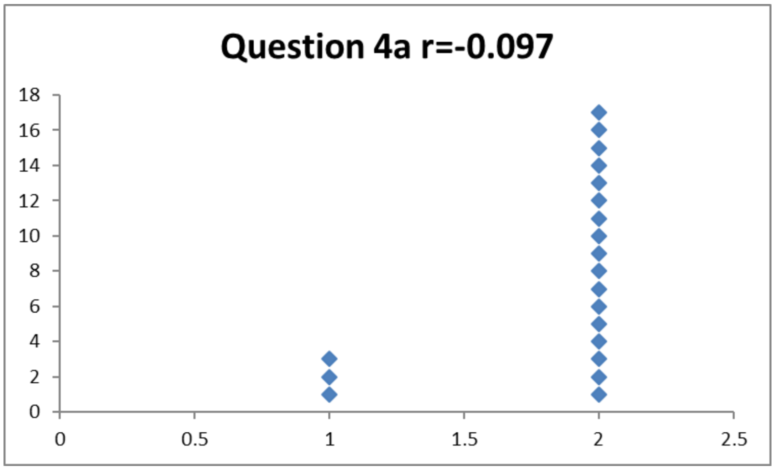
$$R_3 \to R_3 - R_1 \qquad\qquad R_3 \to R_3 + 3R_1$$



4) Consider a 4×6 matrix A that has 3 pivots. (2 points each)

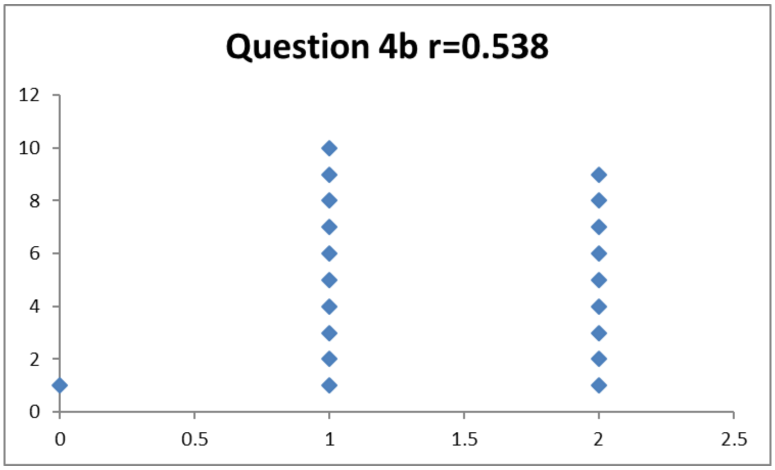
(A) How many solutions does $A\vec{x} = \vec{0}$ have?

Infinitely many, because there is a free variable.



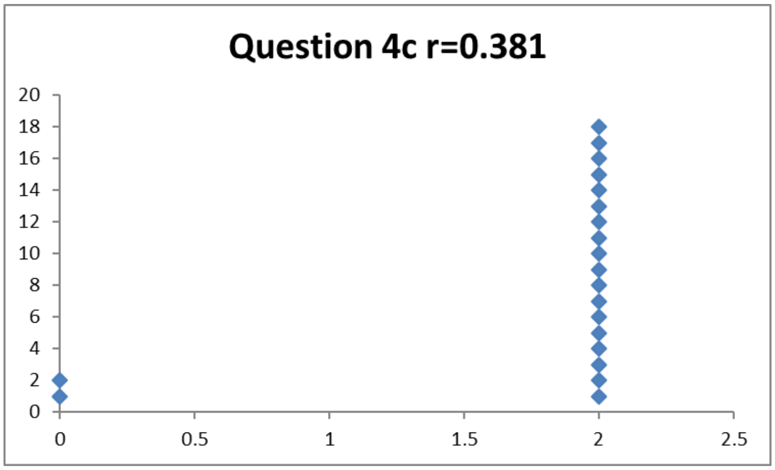
(B) How many free variables does the equation $A\vec{x} = \vec{0}$ have?

Thee free variables, because there are 6 columns each corresponding to one variable. Three of the variables are leading, given by the pivots. The rest are free.



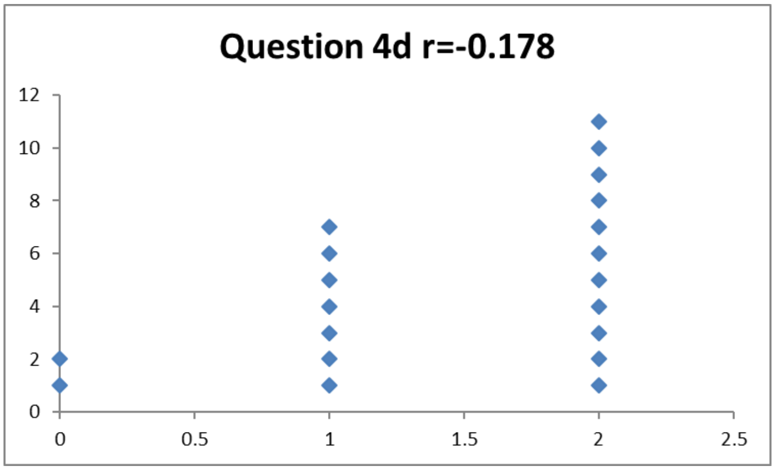
(C) If $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 7 \end{bmatrix}$ has no solutions, how many solutions does it have?

There are no solutions … because, well … there are no solutions.



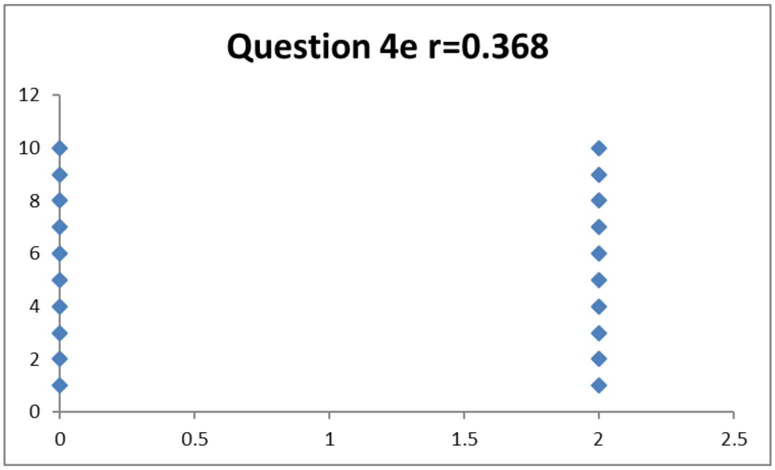
(D) If $A\vec{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 7 \end{bmatrix}$ has a solution, how many solutions does it have?

There are infinitely many solutions, because as soon as there are multiple, there is an entire vector space of solutions.



(E) Does A have an inverse? True or false.

No, it is not square. Only square matrices can have inverses.



5) Find the inverse of the matrix below. (10 points)

$$\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 3 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 3 & 3 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -3 & 3 & -2 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 & 6 & -5 & 3 \\ 0 & 1 & 0 & -3 & 3 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 \end{bmatrix}$$

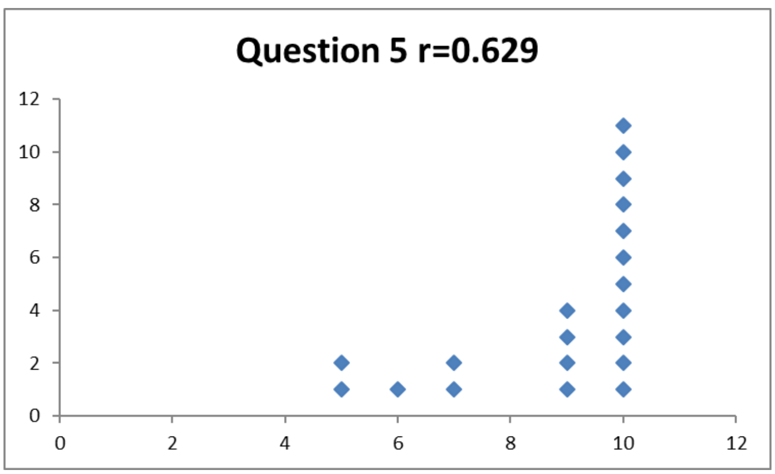
$R_2 \to R_2 - R_1$

$R_1 \to R_1 - 2R_2$, $R_3 \to R_3 - R_2$

$R_1 \to R_1 + 3R_3$, $R_2 \to R_2 - 2R_3$

The inverse matrix is:

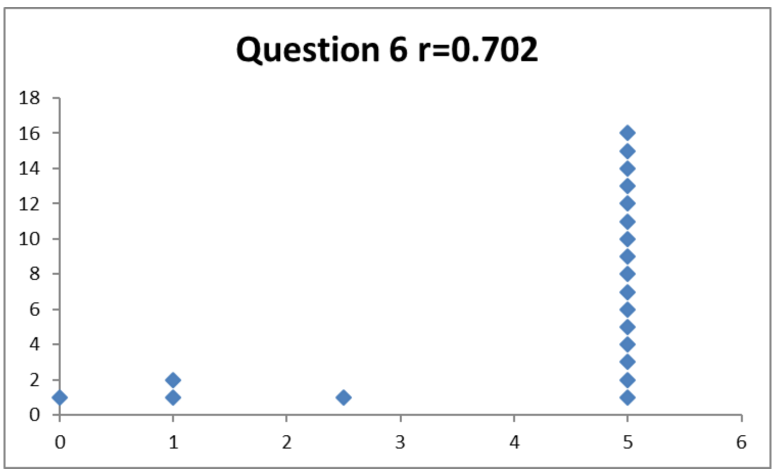
$$\begin{bmatrix} 6 & -5 & 3 \\ -3 & 3 & -2 \\ 1 & -1 & 1 \end{bmatrix}$$



6) Find the product below. (5 points)

$$\begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 5 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 & 0 & 0 & 0 \\ 3 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 3 & 0 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 10 & 0 & 0 & 0 & 0 \\ 17 & 26 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 8 & 0 & 0 \\ 0 & 0 & 4 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 17 \\ 0 & 0 & 0 & 0 & 2 & 9 \end{bmatrix}$$

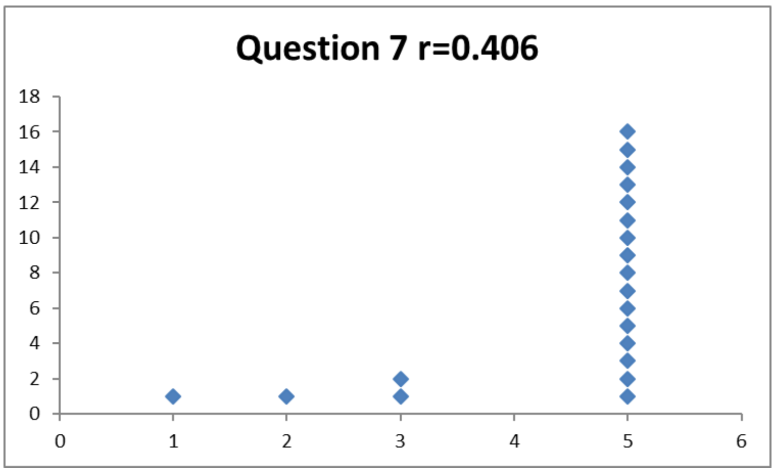


7) Determine whether or not the vectors below are orthogonal. Justify your answer. (5 points)

$$\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} \bullet \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} = -6 + 0 + 3 = -3 \neq 0$$

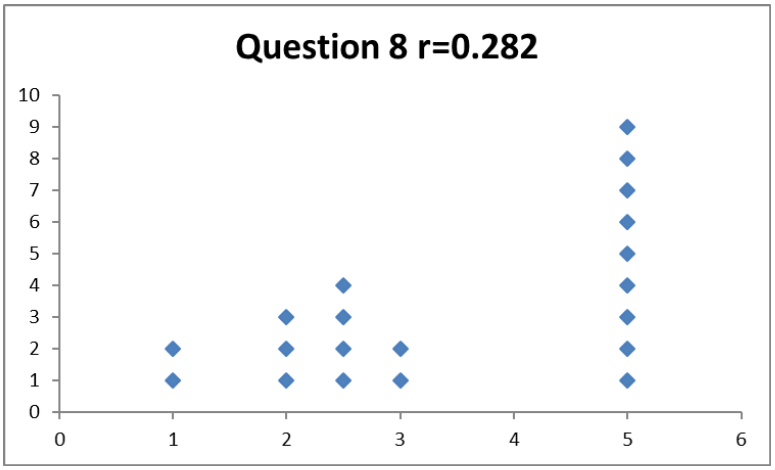
They are not orthogonal because the dot product is nonzero.



8) Below is a matrix equation. Write down the corresponding system of homogeneous equations. (5 points)

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 7 \end{bmatrix}$$

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 0 \\ 4x_1 \qquad\quad + 7x_3 &= 0 \end{aligned}$$

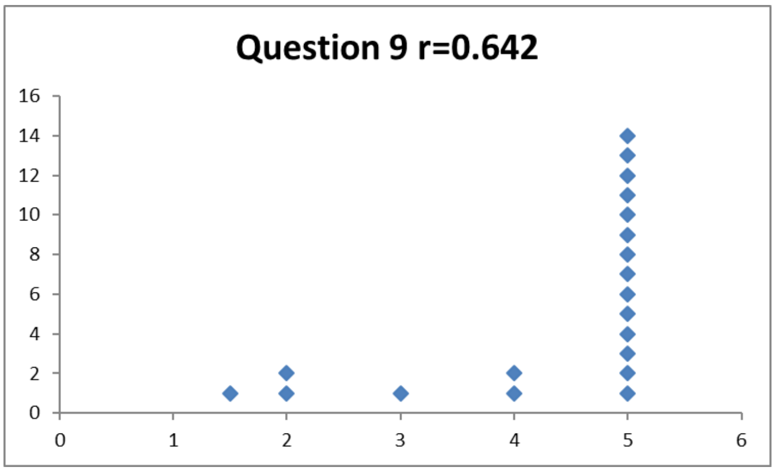


***Whoops, that's actually just a matrix not a matrix equation. Because of the duality, you were still able to solve the problem with the information given. However, if you noticed this issue, 3 bonus points were awarded.

9) Use the formula $||\vec{v}|| \cdot ||\vec{w}|| \cdot \cos(\theta) = \vec{v} \bullet \vec{w}$ to find the angle between the two vectors below. You do not need to simplify your answer. (5 points)

$$\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix}$$

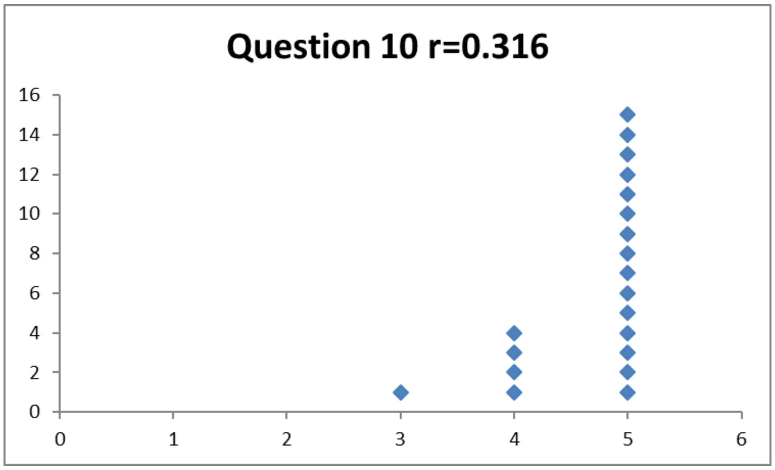
$$cos^{-1}\left(\frac{-6+5}{\sqrt{9+1+16}\sqrt{4+25}}\right)$$



10) Given the two vectors below, find $2\vec{v} - 3\vec{w}$. (5 points)

$$\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \vec{w} = \begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix}$$

$$2\vec{v} - 3\vec{w} = 2\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} - 3\begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 8 \end{bmatrix} - \begin{bmatrix} -6 \\ 15 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 \\ -13 \\ 8 \end{bmatrix}$$

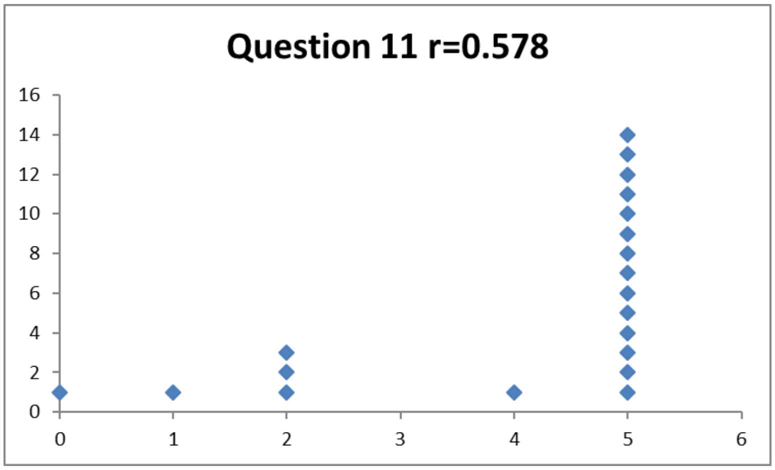


11) Find $\vec{v}^T\vec{w}$, given the two vectors below. (5 points)

$$\vec{v} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \vec{w} = \begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix}$$

$$[3 \quad 1 \quad 4]\begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix} = [-6 + 5 + 0] = [-1]$$

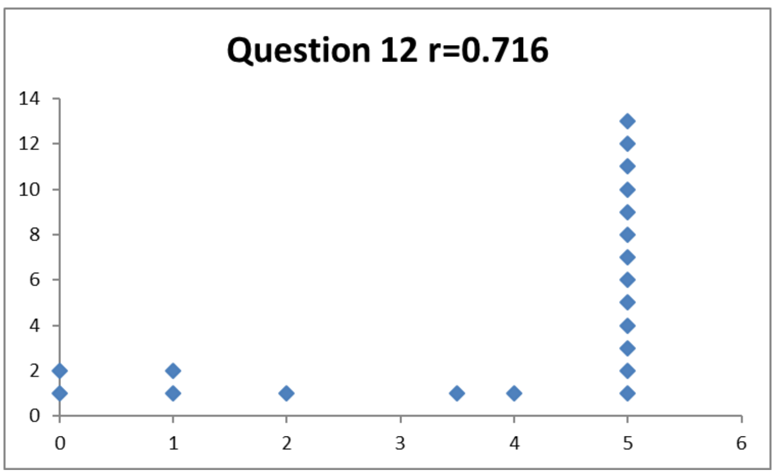
A 1×1 matrix can be treated as a number, so full credit is given for just -1.



12) Given the information below, solve $A\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ (5 points)

$$A = \begin{bmatrix} 4 & 5 & -4 \\ 2 & 4 & -3 \\ -1 & -1 & 1 \end{bmatrix}, A^{-1} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 4 \\ 2 & -1 & 6 \end{bmatrix}$$

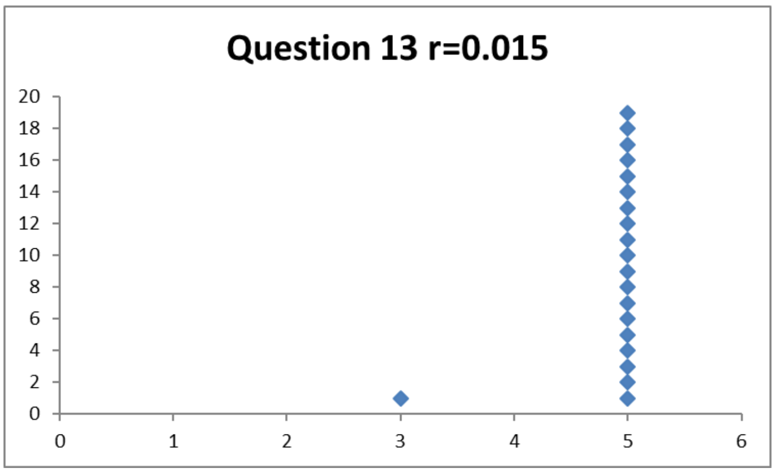
$$\vec{x} = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 4 \\ 2 & -1 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \\ 14 \end{bmatrix}$$



13) Find the length of the vector below. (5 points)

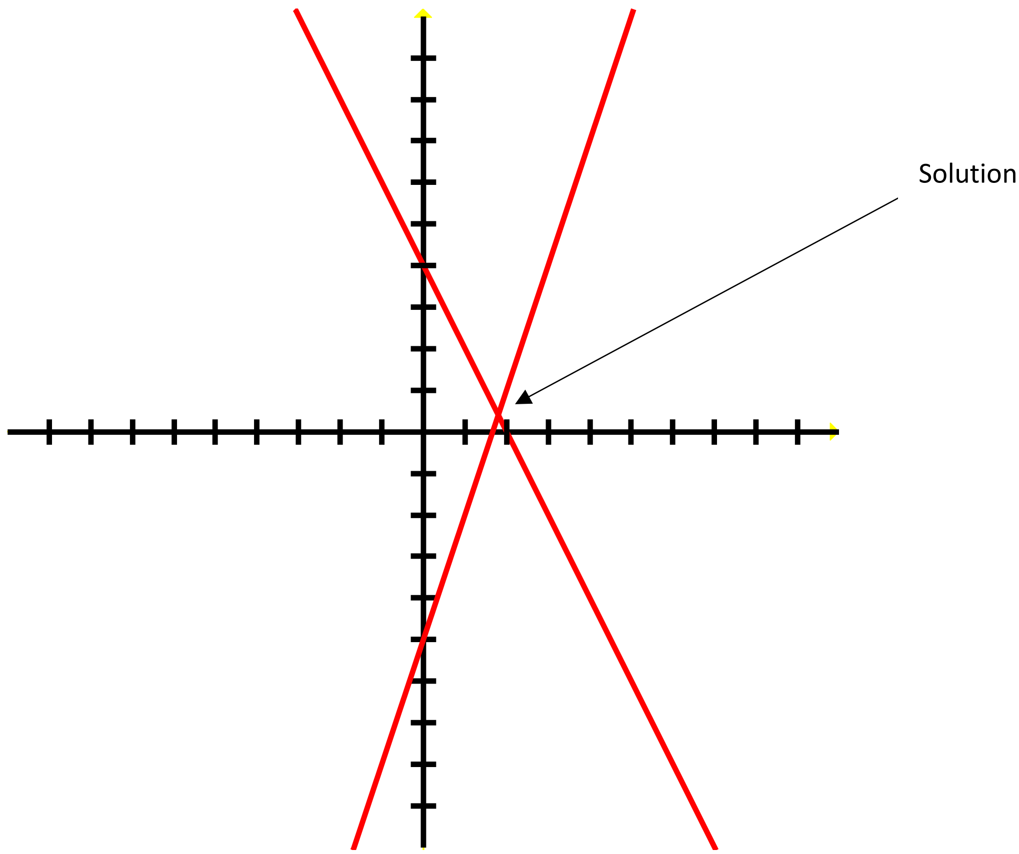
$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 5 \end{bmatrix}$$

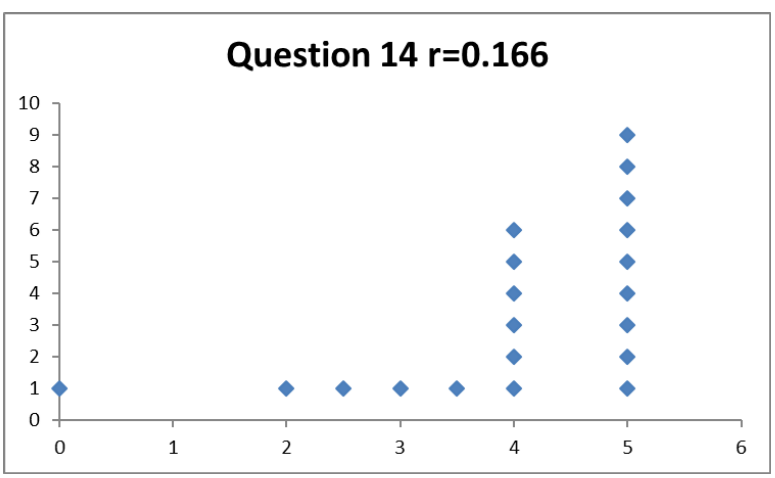
$$\sqrt{1^2 + 2^2 + 0^2 + 5^2} = \sqrt{30}$$



14) Graphically illustrate the solution to the system of equations below. (5 points)

$$2x + y = 4$$
$$3x - y = 5$$





15) Find the transpose of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 0 & 2 & 8 \\ 3 & 4 & 5 & 0 \\ 1 & 0 & 2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 1 \\ 0 & 4 & 0 \\ 2 & 5 & 2 \\ 8 & 0 & 1 \end{bmatrix}$$