Name ____ Test 2, Spring 2020

1) Find the rank of the matrix below. (10 points)

$$\begin{bmatrix} 1 & 0 & 5 & 4 & 0 \\ 0 & 1 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2) A basis B and a vector $\vec{x}$ are given below. Find $[\vec{x}]_S$. (10 points)

$$B = \left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -3 \end{bmatrix} \right\}; \vec{x} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

3) Given the information below, write down a formula for $[\vec{x}]_{B_2}$. You do not need to compute or simplify your answer. (10 points)

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix} \right\} \quad B_2 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix} \right\} \quad \vec{x} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}_{B_1}$$

4) Find the rank of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 6 & 30 \end{bmatrix}$$

5) Compute the following. (10 points)

$$\begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 7 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{vmatrix}$$

6) Suppose A is a 6×13 matrix. In row reduced echelon form, it has 5 pivots. (10 points)

(A) What is the dimension of the null space of A?

(B) What is the dimension of the range of the corresponding linear transformation?

(C) What is the dimension of the solution set to the corresponding homogeneous linear transformation?

(D) Is the corresponding linear transformation onto? Why or why not?

(E) What is the rank of A?

7) Is $\begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$ in the span $\left(\left\{ \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix} \right\} \right)$? Why or why not? (5 points)

8) What is the column space of the matrix below? Avoid using redundant vectors when possible. (5 points)

$$\begin{bmatrix} 1 & 4 & 3 & 7 \\ 2 & 6 & 4 & 12 \\ -1 & -1 & 0 & -4 \end{bmatrix}$$

9) Is the function below a linear transformation? Why or why not? (5 points)

$$T: \mathbb{R}^2 \to \mathbb{R}^2$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} 2x_1 + 3x_2 \\ x_1 - x_2 \end{bmatrix}$$

10) Is linear transformation below one-to-one? Why or why not? (5 points)

$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 7 & 31 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

11) What is the kernel of the linear transformation below? (10 points)

$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 7 & 31 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

12) Use Cramer's Rule to find a formula for x_3 in the system of equations below. (5 points)

$$3x_1 + 2x_2 + 4x_3 = 7$$
$$5x_1 + 5x_2 + 2x_3 = 2$$
$$6x_1 + 9x_2 + 1x_3 = 4$$

13) Suppose the system of equations $A\vec{x} = \vec{b}$ does not have a solution. A is a 5×8 matrix. (10 points)

(A) What is the maximum number of free variables?

(B) What is the minimum number of free variables?

(C) When in row reduced echelon form, what is the maximum number of zero rows?

(D) When in row reduced echelon form, what is the minimum number of zero rows?

(E) What is the maximum dimension of the range of the corresponding linear transformation?

The following matrices have been row reduced as shown. They might be helpful during the test. You may tear this sheet off.

$$\begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 7 & 31 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 6 & 30 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 4 \\ 5 & 5 & 2 \\ 6 & 9 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & | & 1 \\ 3 & 2 & | & 5 \\ 0 & 6 & | & 7 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 3 & 7 \\ 2 & 6 & 4 & 12 \\ -1 & -1 & 0 & -4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$