Name ____ Test 2, Spring 2020

1) Find the rank of the matrix below. (10 points)

$$\begin{bmatrix} 1 & 0 & 5 & 4 & 0 \\ 0 & 1 & 3 & 5 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3, because there are 3 pivots.

2) A basis B and a vector $\vec{x}$ are given below. Find $[\vec{x}]_S$. (10 points)

$$B = \left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 4 \\ -3 \end{bmatrix} \right\}; \vec{x} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 1 & 0 & 4 \\ 1 & 2 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \\ -1 \end{bmatrix}$$

3) Given the information below, write down a formula for $[\vec{x}]_{B_2}$. You do not need to compute or simplify your answer. (10 points)

$$B_1 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 0 \end{bmatrix} \right\} \quad B_2 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 7 \end{bmatrix} \right\} \quad \vec{x} = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}_{B_1}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 3 & 1 & 7 \end{bmatrix}^{-1} \begin{bmatrix} 1 & -1 & 5 \\ 0 & 1 & 5 \\ 3 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$$

4) Find the rank of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 6 & 30 \end{bmatrix}$$

2

$$\begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 6 & 30 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

5) Compute the following. (10 points)

$$\begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 7 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 \\ 1 & 7 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{vmatrix} = 1 \begin{vmatrix} 0 & 1 & 2 \\ 7 & 1 & 0 \\ 1 & 2 & 0 \end{vmatrix} - 2 \begin{vmatrix} 0 & 0 & 1 \\ 1 & 7 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 1 \cdot 2 \cdot \begin{vmatrix} 7 & 1 \\ 1 & 2 \end{vmatrix} - 2 \cdot 1 \begin{vmatrix} 1 & 7 \\ 0 & 1 \end{vmatrix} = 2(14-1) - 2(1) = 24$$

6) Suppose A is a 6×13 matrix. In row reduced echelon form, it has 5 pivots. (10 points)

(A) What is the dimension of the null space of A?

$$13 - 5 = 8$$

(B) What is the dimension of the range of the corresponding linear transformation?

$$5$$

(C) What is the dimension of the solution set to the corresponding homogeneous linear transformation?

$$8$$

(D) Is the corresponding linear transformation onto? Why or why not?

No, because in row reduced echelon form, there is a zero row.

(E) What is the rank of A?

$$5$$

7) Is $\begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$ in the span$\left(\left\{\begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 6 \end{bmatrix}\right\}\right)$? Why or why not? (5 points)

No, because the column corresponding to $\begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$ has a pivot: $\begin{bmatrix} 2 & 4 & | & 1 \\ 3 & 2 & | & 5 \\ 0 & 6 & | & 7 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8) What is the column space of the matrix below? Avoid using redundant vectors when possible. (5 points)

$$\begin{bmatrix} 1 & 4 & 3 & 7 \\ 2 & 6 & 4 & 12 \\ -1 & -1 & 0 & -4 \end{bmatrix}$$

$$span\left(\left\{\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 6 \\ -1 \end{bmatrix}\right\}\right)$$

$$\begin{bmatrix} 1 & 4 & 3 & 7 \\ 2 & 6 & 4 & 12 \\ -1 & -1 & 0 & -4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

9) Is the function below a linear transformation? Why or why not? (5 points)

$$T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$$
$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \mapsto \begin{bmatrix} 2x_1 + 3x_2 \\ x_1 - x_2 \end{bmatrix}$$

Yes, because it can be represented by matrix multiplication: $\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

10) Is linear transformation below one-to-one? Why or why not? (5 points)

$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 7 & 31 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

No, because the row reduced matrix has a column without a pivot.

$$\begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 7 & 31 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

11) What is the kernel of the linear transformation below? (10 points)

$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \mapsto \begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 7 & 31 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$span\left(\left\{\begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix}\right\}\right)$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 7 & 31 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

12) Use Cramer's Rule to find a formula for x_3 in the system of equations below. (5 points)

$$3x_1 + 2x_2 + 4x_3 = 7$$
$$5x_1 + 5x_2 + 2x_3 = 2$$
$$6x_1 + 9x_2 + 1x_3 = 4$$

$$x_3 = \frac{\begin{vmatrix} 3 & 2 & 7 \\ 5 & 5 & 2 \\ 6 & 9 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 2 & 4 \\ 5 & 5 & 2 \\ 6 & 9 & 1 \end{vmatrix}}$$

13) Suppose the system of equations $A\vec{x} = \vec{b}$ does not have a solution. A is a 5×8 matrix. (10 points)

In row reduced echelon form, there is a row of zeroes. That means the rank is at most 4.

(A) What is the maximum number of free variables?

8

(B) What is the minimum number of free variables?

4

(C) When in row reduced echelon form, what is the maximum number of zero rows?

5

(D) When in row reduced echelon form, what is the minimum number of zero rows?

1

(E) What is the maximum dimension of the range of the corresponding linear transformation?

4

The following matrices have been row reduced as shown. They might be helpful during the test. You may tear this sheet off.

$$\begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 7 & 31 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 5 \\ 7 & 4 & 25 \\ 8 & 6 & 30 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 4 \\ 5 & 5 & 2 \\ 6 & 9 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & 4 & 1 \\ 3 & 2 & 5 \\ 0 & 6 & 7 \end{array}\right] \sim_R \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 3 & 7 \\ 2 & 6 & 4 & 12 \\ -1 & -1 & 0 & -4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$