

**Instructions:**

- You will need to submit two things:
  - Your answers must be submitted on Blackboard under **“Test 3”**.
  - Your supporting work must be submitted shortly afterward under **“Test 3 Work”**. Your work must match the answers you submit.
- The test opened at 10:50am. You have a half hour to start the test. Once you start you have 90 minutes to complete the test. (35 for the practice test)
- This test is open notes, book, internet, etc. You may use any static resources you like, but may not ask any person for assistance. As such, significantly more points will be weighted on the explanations and work than the answers themselves. Seriously, very clearly show your work, if it looks like a computer algebra system gave you an answer and you don’t understand what it means, you will not receive credit.

1) Below is a basis of a vector space. Use the process we learned to turn it into an orthogonal basis.

Show your work. (20 points)

$$\left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \\ 1 \end{bmatrix} \right\}$$

2) Below is a matrix. Find the eigenvalues and corresponding eigenspaces. Show your work. (20 points)

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 2 & 4 & 2 \end{bmatrix}$$

3) Diagonalize the matrix below and express your answer as an equation. Show your work. (20 points)

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -3 & -2 \\ 2 & 4 & 2 \end{bmatrix}$$

4) A  $3 \times 3$  matrix has eigenvalues 1, 2, and 4. What is the rank of  $A$ ? (4 points)

5) A  $3 \times 3$  matrix has eigenvalues 0, 1, and 3. What is the rank of  $A$ ? (4 points)

6) A  $3 \times 3$  matrix has eigenvalues 0, 0, and 5.  $A$  is diagonalizable. What is the rank of  $A$ ? (4 points)

7) A  $3 \times 3$  matrix has eigenvalues 0, 0, and 5.  $A$  is not diagonalizable. What is the rank of  $A$ ? (4 points)

8) A  $3 \times 3$  matrix has eigenvalues 0, 1, and 3. How many solutions are there to  $A\vec{x} = \vec{0}$ ? (4 points)

9) Find the determinant of the matrix below. Show your work. (10 points)

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & 3 & -1 \\ 5 & 2 & -3 \end{bmatrix}$$

10) Row reduce the matrix below into reduced row echelon form. Show your work. (10 points)

$$\begin{bmatrix} 1 & 2 & 4 & 2 \\ 0 & 3 & -9 & 3 \\ 5 & 2 & -4 & 0 \end{bmatrix}$$