

Name \_\_\_\_\_ Test 1, Fall 2021

1) Multiply the two matrices below or state why they cannot be multiplied. (15 points)

$$\begin{bmatrix} 1 & 6 \\ 3 & 2 \\ 7 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 & 3 \\ 2 & -1 & 5 \end{bmatrix}$$

2) Find the null space of the matrix below. (15 points)

$$\begin{bmatrix} 0 & 1 & 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 6 & 0 & -4 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

3) Reduce the matrix below to reduced row echelon form. (15 points)

$$\begin{bmatrix} 0 & 0 & 3 & 2 & 1 \\ 1 & 2 & 9 & 6 & 3 \\ 2 & 4 & 6 & 4 & 2 \\ 0 & 0 & 12 & 4 & 4 \end{bmatrix}$$

4) Answer the questions below (3 points each)

(A) Let  $A$  be a  $3 \times 3$  matrix such that  $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}$  has a unique solution.

Is  $A$  a product of elementary matrices?

(B) Let  $A$  be a  $5 \times 7$  matrix. When row reduced, it has 4 pivots. What is the dimension of the column space?

(C) Let  $A$  be a  $6 \times 4$  matrix. When row reduced, it has 3 pivots.

How many solutions can  $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$  have?

(D) Let  $A$  be a  $4 \times 3$  matrix such that  $A\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$  has a unique solution, but  $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2 \end{bmatrix}$  has no solutions. What is the rank of  $A$ ?

(E) Let  $A$  be a  $12 \times 7$  matrix whose row space has dimension 4. When row reduced, how many pivots does it have?

5) Find the inverse of the matrix below. (10 points)

$$\begin{bmatrix} 9 & 18 & 27 \\ 2 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix}$$

The following row reduction may or may not be useful for the problems on this page.

$$\begin{bmatrix} 1 & 2 & 4 & 1 \\ 0 & 3 & 3 & 1 \\ 2 & 1 & 5 & 1 \\ 1 & -5 & -3 & 2 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

6) Determine whether or not the collection below is a vector space. Why? (5 points)

$$\left\{ \begin{bmatrix} x_1 + 2x_2 + 4x_3 + x_4 \\ 3x_2 + 3x_3 + x_4 \\ 2x_1 + x_2 + 5x_3 + x_4 \\ x_1 - 5x_2 - 3x_3 + 2x_4 \end{bmatrix} : x_1, x_2, x_3, x_4 \in \mathbb{R} \right\}$$

7) Can  $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$  be written as a linear combination of  $\begin{bmatrix} 1 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ ,  $\begin{bmatrix} 2 \\ 3 \\ 1 \\ -5 \end{bmatrix}$ , and  $\begin{bmatrix} 4 \\ 3 \\ 5 \\ -3 \end{bmatrix}$ . Why? (5 points)

8) Find the row space of the matrix below. Do not include redundant vectors. (5 points)

$$\begin{bmatrix} 1 & 2 & 4 & 1 \\ 0 & 3 & 3 & 1 \\ 2 & 1 & 5 & 1 \\ 1 & -5 & -3 & 2 \end{bmatrix}$$

9) Use the information below to solve the system of equations below. You do not need to simplify your answer. (5 points)

$$\begin{bmatrix} 12 & 7 & 3 \\ 20 & 13 & 6 \\ 3 & 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 & 3 \\ -2 & 3 & -12 \\ 1 & -3 & 16 \end{bmatrix}$$

$$12x + 7y + 3z = 2$$

$$20x + 13y + 6z = 1$$

$$3x + 2y + z = 4$$

10) How many solutions does the equation below have? (5 points)

$$\begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$$

11) Find the product below. (5 points)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 & 2 & 1 \\ 3 & 4 & 4 & 4 & 3 \\ 5 & 6 & 6 & 6 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 2 & 3 \end{bmatrix}$$