Name ____ Test 3, Fall 2021

1) Given the matrix below, find its eigenspaces. Circle or box your answer(s). (15 points)

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

2) Find the eigenvalues of the linear transformation below. (5 points)

$$T: \mathbb{R}^4 \to \mathbb{R}^4$$
$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \mapsto \begin{bmatrix} 2x_1 \\ 3x_2 \\ 5x_3 \\ x_4 \end{bmatrix}$$

3) Given the basis below, find an orthogonal version of the basis. (10 points)

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -13 \end{bmatrix} \right\}$$

4) Answer the questions below (3 points each)

(A) Let A be a 3×3 matrix with eigenvalues $0, 1$, and 4. What is $|A|$?

(B) Let A be a 3×3 matrix with eigenvalues $0, 1$, and 1 again. What is the dimension of the null space of A?

(C) Let $T: \mathbb{R}^6 \to \mathbb{R}^{10}$ be a one-to-one linear transformation. What is the dimension of $\ker(T)$.

(D) Let A be a 12×4 matrix whose columns are linearly independent. Is the corresponding linear transformation onto?

(E) Let $T: \mathbb{R}^5 \to \mathbb{R}^5$ be a linear transformation that is onto. Is it one-to-one?

5) Find the null space of the matrix below. (10 points)

$$\begin{bmatrix} 2 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6) Consider the basis below and the coordinate vector $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}_B$. Find a formula for this same vector, represented in the standard coordinates. (10 points)

$$B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} \right\}$$

7) Consider the two bases below and the vector $[\vec{x}]_{B_2} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}_{B_2}$. Find a formula for $[\vec{x}]_{B_1}$. (10 points)

$$B_1 = \left\{ \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \end{bmatrix} \right\} \quad B_1 = \left\{ \begin{bmatrix} 0 \\ 3 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \end{bmatrix} \right\}$$

8) Find the diagonalization of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$$

9) Find a formula for $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}^{1000}$ that involves no more than 5 matrix multiplications. (5 points)

10) If it is known that $T\left(\begin{bmatrix}1\\0\end{bmatrix}\right) = \begin{bmatrix}2\\4\end{bmatrix}$ and $T\left(\begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}6\\3\end{bmatrix}$, find the rule for T. (5 points)

11) Find a formula for the angle between the vectors $\begin{bmatrix}2\\4\end{bmatrix}$ and $\begin{bmatrix}6\\5\end{bmatrix}$. (5 points)

12) Write down $\operatorname{span}\left(\left\{\begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 5 \end{bmatrix}\right\}\right)$ using set builder notation. (5 points)