1) Find the product below. (15 points)

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 4 & 0 \\ 3 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & 3 \\ 0 & 3 & -1 \end{bmatrix}$$

2) Row reduce the matrix below to reduced echelon form. (15 points)

$$\begin{bmatrix} 6 & 12 & 6 & 18 \\ 2 & 4 & 2 & 5 \\ 5 & 10 & 2 & 5 \end{bmatrix}$$

3) Find the null space of the matrix below. (15 points)

$$\begin{bmatrix} 1 & 3 & 0 & -4 \\ 1 & 4 & 0 & 4 \end{bmatrix}$$

4) Answer the following questions. (3 points each)

A) Let A be a 5×5 invertible matrix. How many solutions can $A\vec{x} = \vec{0}$ have?

B) Let A be a 3×3 matrix such that $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$ has no solutions. How many solutions can $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$ have?

C) Let $A\vec{x} = \vec{b}$ be a system of 3 equations in 3 variables with a unique solution. What is the row space of A?

D) Let A be a 6×4 matrix, which when row reduced has 3 pivots. How many solutions can $A\vec{x} = \vec{b}$ with $\vec{b} \neq \vec{0}$ have?

E) Let A be a 6×4 matrix, which when row reduced has 4 pivots. How many solutions can $A\vec{x} = \vec{b}$ with $\vec{b} \neq \vec{0}$ have?

5) For each of the following, is it true or false that it is possible to multiply the matrices given? (1 point each)

T or F   A) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

T or F   B) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

T or F   C) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

T or F   D)   The product AB where A is 2×3 and B is 4×2

T or F   E)   The product AB where A is 2×3 and B is 4×5

6) Find the product below. (5 points)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 & 2 & 3 \\ 3 & 4 & 4 & 5 & 5 \\ 6 & 6 & 4 & 4 & 5 \\ 5 & 3 & 3 & 2 & 2 \\ 1 & 2 & 2 & 1 & 3 \end{bmatrix}$$

For the problems on this page, you might be interested in the following fact.

$$\begin{bmatrix} 2 & 4 & -4 & 1 & 1 & 1 & 1 \\ 1 & 2 & -2 & 0 & 2 & 0 & 4 \\ 1 & 2 & -2 & 3 & 2 & 3 & 1 \\ 4 & 8 & -8 & 2 & 3 & 2 & 4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

7) Express the span below in set builder notation. Do not include redundant vectors. (10 points)

$$span\left(\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 8 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \\ -2 \\ -8 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 1 \\ 4 \end{bmatrix} \right\}\right)$$

8) Solve the system of equations below. (10 points)

$$\begin{aligned} 2x_1 + 4x_2 - 4x_3 + x_4 + x_5 + x_6 &= 1 \\ x_1 + 2x_2 - 2x_3 \qquad + 2x_5 \qquad &= 4 \\ x_1 + 2x_2 - 2x_3 + 3x_4 + 2x_5 + 3x_6 &= 1 \\ 4x_1 + 8x_2 - 8x_3 + 2x_4 + 3x_5 + 2x_6 &= 4 \end{aligned}$$

9) On the axis provided, illustrate the length of the vector below. (5 points)

$$\vec{v} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}, \ \|\vec{v}\| = 5$$
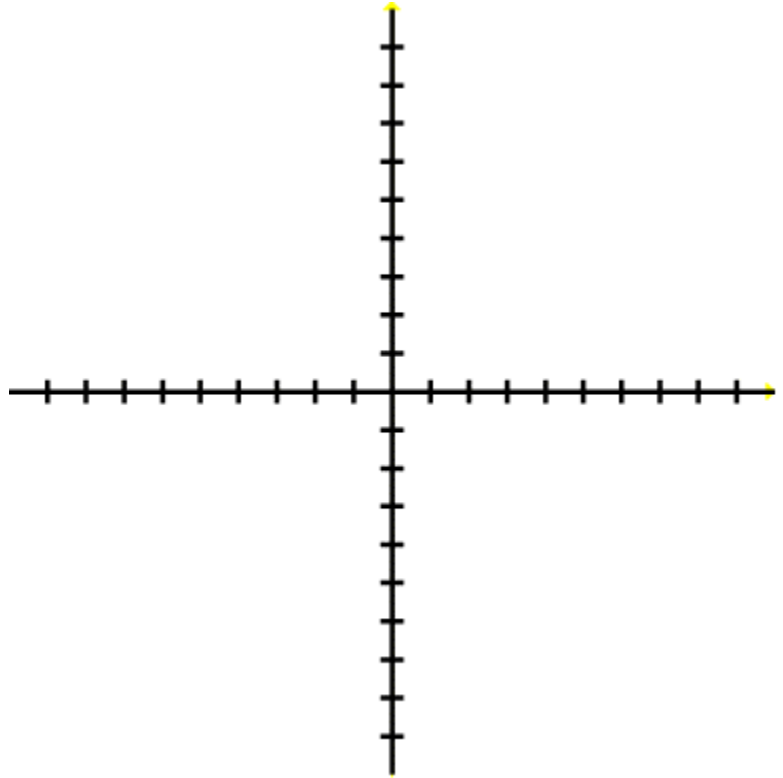
10) What is the inverse of the matrix below? (5 points)

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$