Name ____ Test 1, Spring 2021

1) Find the product below. (15 points)

$$\begin{bmatrix} 2 & 3 & -2 \\ 1 & 4 & 0 \\ 3 & 0 & 2 \end{bmatrix}\begin{bmatrix} 1 & 5 & 0 \\ 2 & 4 & 3 \\ 0 & 3 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 16 & 11 \\ 9 & 21 & 12 \\ 3 & 21 & -2 \end{bmatrix}$$

2) Row reduce the matrix below to reduced echelon form. (15 points)

$$\begin{bmatrix} 6 & 12 & 6 & 18 \\ 2 & 4 & 2 & 5 \\ 5 & 10 & 2 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 12 & 6 & 18 \\ 2 & 4 & 2 & 5 \\ 5 & 10 & 2 & 5 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 4 & 2 & 5 \\ 5 & 10 & 2 & 5 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 0 & 0 & -1 \\ 5 & 10 & 2 & 5 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -3 & -10 \end{bmatrix}$$

$$R_1 \to \tfrac{1}{6}R_1 \qquad R_2 \to R_2 - 2R_1 \qquad R_3 \to R_3 - 5R_1$$

$$\sim_R \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 0 & -3 & -10 \\ 0 & 0 & 0 & -1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 1 & 3 \\ 0 & 0 & 1 & 3.\overline{3} \\ 0 & 0 & 0 & -1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 0 & -0.\overline{3} \\ 0 & 0 & 1 & 3.\overline{3} \\ 0 & 0 & 0 & -1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 0 & -0.\overline{3} \\ 0 & 0 & 1 & 3.\overline{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3 \qquad R_2 \to -\tfrac{1}{3}R_2 \qquad R_1 \to R_1 - R_2 \qquad R_3 \to -R_3$$

$$\sim_R \begin{bmatrix} 1 & 2 & 0 & -0.\overline{3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 - 3.\overline{3}R_3 \qquad R_1 \to R_1 + 0.\overline{3}R_3$$

Note that you can avoid fractions on this if you go out of order. Just be careful not to mess up parts you've already finished!

3) Find the null space of the matrix below. (15 points)

$$\begin{bmatrix} 1 & 3 & 0 & -4 \\ 1 & 4 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 0 & -4 \\ 1 & 4 & 0 & 4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 3 & 0 & -4 \\ 0 & 1 & 0 & 8 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 & -28 \\ 0 & 1 & 0 & 8 \end{bmatrix}$$

$x_3 \in \mathbb{R}$
$x_4 \in \mathbb{R}$
$x_2 = -8x_4$
$x_1 = 28x_4$

The null space is:

$$span\left(\left\{\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 28 \\ -8 \\ 0 \\ 1 \end{bmatrix}\right\}\right)$$

4) Answer the following questions. (3 points each)

A) Let A be a 5×5 invertible matrix. How many solutions can $A\vec{x} = \vec{0}$ have?

1

B) Let A be a 3×3 matrix such that $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$ has no solutions. How many solutions can $A\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 6 \end{bmatrix}$ have?

0 or ∞

C) Let $A\vec{x} = \vec{b}$ be a system of 3 equations in 3 variables with a unique solution. What is the row space of A?

$\mathbb{R}^3$

D) Let A be a 6×4 matrix, which when row reduced has 3 pivots. How many solutions can $A\vec{x} = \vec{b}$ with $\vec{b} \neq \vec{0}$ have?

0 or ∞

E) Let A be a 6×4 matrix, which when row reduced has 4 pivots. How many solutions can $A\vec{x} = \vec{b}$ with $\vec{b} \neq \vec{0}$ have?

0 or 1

5) For each of the following, is it true or false that it is possible to multiply the matrices given? (1 point each)

T or F A) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

T or F B) $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

T or F C) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$

T or F D) The product AB where A is 2×3 and B is 4×2

T or F E) The product AB where A is 2×3 and B is 4×5

6) Find the product below. (5 points)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 & 2 & 3 \\ 3 & 4 & 4 & 5 & 5 \\ 6 & 6 & 4 & 4 & 5 \\ 5 & 3 & 3 & 2 & 2 \\ 1 & 2 & 2 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 4 & 5 & 5 \\ 6 & 6 & 4 & 4 & 5 \\ 15 & 9 & 9 & 6 & 6 \\ 1 & 2 & 2 & 1 & 3 \end{bmatrix}$$

For the problems on this page, you might be interested in the following fact.

$$\begin{bmatrix} 2 & 4 & -4 & 1 & 1 & 1 & 1 \\ 1 & 2 & -2 & 0 & 2 & 0 & 4 \\ 1 & 2 & -2 & 3 & 2 & 3 & 1 \\ 4 & 8 & -8 & 2 & 3 & 2 & 4 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 2 & -2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

7) Express the span below in set builder notation. Do not include redundant vectors. (10 points)

$$span\left(\left\{\begin{bmatrix} 2 \\ 1 \\ 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 4 \\ 2 \\ 2 \\ 8 \end{bmatrix}, \begin{bmatrix} -4 \\ -2 \\ -2 \\ -8 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 1 \\ 4 \end{bmatrix}\right\}\right)$$

$$\left\{\begin{bmatrix} 2 \\ 1 \\ 1 \\ 4 \end{bmatrix} x_1 + \begin{bmatrix} 1 \\ 0 \\ 3 \\ 2 \end{bmatrix} x_2 + \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix} x_3 : x_1, x_2, x_3 \in \mathbb{R}\right\}$$

8) Solve the system of equations below. (10 points)

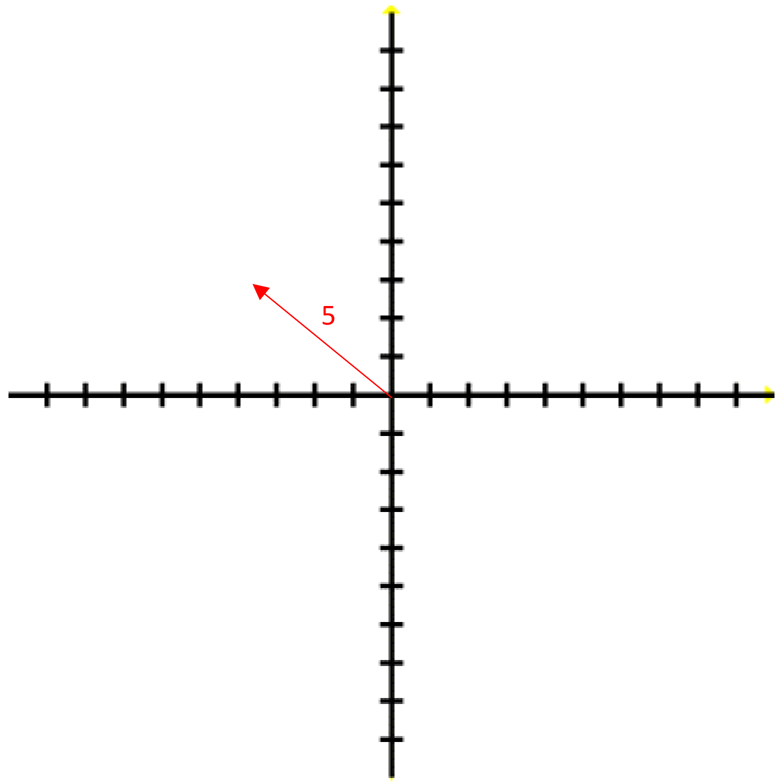
$$\begin{aligned} 2x_1 + 4x_2 - 4x_3 \quad + x_4 \quad + x_5 \quad + x_6 &= 1 \\ x_1 + 2x_2 - 2x_3 \qquad\qquad + 2x_5 \qquad\quad &= 4 \\ x_1 + 2x_2 - 2x_3 + 3x_4 + 2x_5 + 3x_6 &= 1 \\ 4x_1 + 8x_2 - 8x_3 + 2x_4 + 3x_5 + 2x_6 &= 4 \end{aligned}$$

$x_2, x_3, x_6, x_7 \in \mathbb{R}$
$x_1 = -2x_2 + 2x_3$
$x_4 = -x_6 + x_7$
$x_5 = -2x_7$

$$\left\{\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 2 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 1 \end{bmatrix} x_6 + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 2 \\ 0 \end{bmatrix} : x_2, x_3, x_6, x \in \mathbb{R}\right\}$$

9) On the axis provided, illustrate the length of the vector below. (5 points)

$\vec{v} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$, $\|\vec{v}\| = 5$



10) What is the inverse of the matrix below? (5 points)

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}^{-1} = \frac{1}{2-12}\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix} = \frac{-1}{10}\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$$