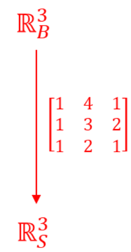
Name ____ Test 2, Spring 2021

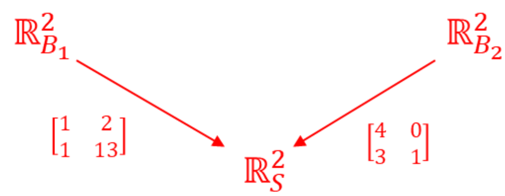
1) Given the basis $B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}$ and $\vec{x}_S = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_S$, find a formula for $[\vec{x}]_B$. (10 points)

$$\begin{bmatrix} 1 & 4 & 1 \\ 1 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$



2) Given the bases $B_1 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 13 \end{bmatrix} \right\}$ and $B_2 = \left\{ \begin{bmatrix} 4 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$, find a formula for the change of basis matrix that converts vectors from basis B_1 into vectors from basis B_2. (10 points)

$$[I]_{B_1}^{B_2} = \begin{bmatrix} 4 & 0 \\ 3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 2 \\ 1 & 13 \end{bmatrix}$$



3) Find the determinant of the matrix below. (15 points)

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & -4 \\ 3 & 4 & 0 & 2 \end{bmatrix}$$

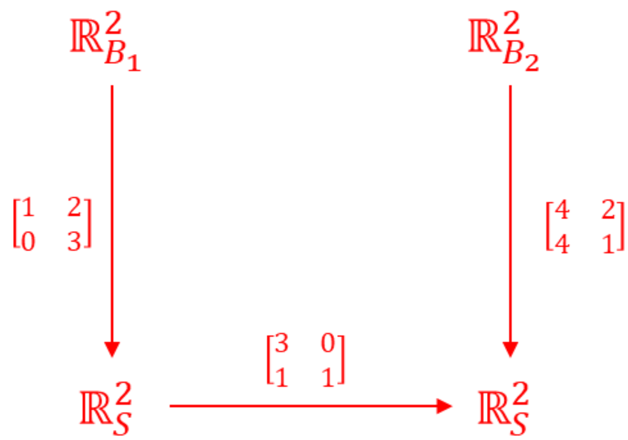
$$\begin{vmatrix} 1 & 2 & 0 & 3 \\ 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & -4 \\ 3 & 4 & 0 & 2 \end{vmatrix} = 1\begin{vmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 3 & 4 & 2 \end{vmatrix} = \begin{vmatrix} 3 & 5 \\ 4 & 2 \end{vmatrix} - 2\begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix} + 3\begin{vmatrix} 1 & 3 \\ 3 & 4 \end{vmatrix}$$

$$= (6 - 20) - 2(2 - 15) + 3(4 - 9) = -14 + 26 - 15 = -3$$

4) Given the linear transformation $T: \mathbb{R}^2_S \to \mathbb{R}^2_S$ given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}_S\right) = \begin{bmatrix} 3x_1 \\ x_1 + x_2 \end{bmatrix}_S$ and the bases below, find a formula for $\left[T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}_{B_1}\right)\right]_{B_2}$. (10 points)

$$B_1 = \left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}\right\}$$
$$B_2 = \left\{\begin{bmatrix} 4 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right\}$$

$$\left[T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}_{B_1}\right)\right]_{B_2} = \begin{bmatrix} 4 & 2 \\ 4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 3 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$



5) Answer the following questions. (3 points each)

A) Let A be a 3×3 matrix and assume that it has rank 2. How many solutions does $A\vec{x} = \vec{0}$ have?

∞

B) Let A be a 3×4 matrix and assume that the corresponding linear transformation T is not onto. What is the minimum dimension of the null space of A?

2

C) Let A be a 3×7 matrix. Assume that the dimension of the row space is 3. What is the dimension of the column space?

3

D) Consider a system of 5 equations in 3 variables. Assume there are infinitely many solutions. If A is the matrix representing this system, what are the possible values for the rank of A?

0, 1, or 2.

E) Let A be a 6×6 matrix and T the corresponding linear transformation. If $\dim(ker(T)) = 2$, what is the rank of A?

4

6) Find the product below. (10 points)

$$\begin{bmatrix}1\\2\end{bmatrix}\begin{bmatrix}3 & 4 & 5\end{bmatrix}$$

$$\begin{bmatrix}3 & 4 & 5\\6 & 8 & 10\end{bmatrix}$$

7) Let $f(x) = 2x + 3$, find $f\left(\begin{bmatrix}1 & 2\\0 & 2\end{bmatrix}\right)$. (5 points)

$$f\left(\begin{bmatrix}1 & 2\\0 & 2\end{bmatrix}\right) = 2\begin{bmatrix}1 & 2\\0 & 2\end{bmatrix} + 3\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}5 & 4\\0 & 7\end{bmatrix}$$

8) Given the information below, find a formula for $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$. (5 points)

$$\begin{bmatrix} 1 & 2 \\ 4 & 7 \end{bmatrix}^{-1} = \begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix}$$

$$\begin{aligned} x_1 + 2x_2 &= 5 \\ 4x_1 + 7x_2 &= 7 \end{aligned}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -7 & 2 \\ 4 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

9) Find a basis for the vector space below. (5 points)

$$span\left(\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}\right\}\right)$$

$$B = \left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\right\}$$

10) Given the information below, find $T\left(\begin{bmatrix}1\\2\end{bmatrix}\right)$. (10 points)

$$T: \mathbb{R}^2 \to \mathbb{R}^3$$

$$[T] = \begin{bmatrix}2 & 0\\1 & 0\\4 & 3\end{bmatrix}$$

$$T\left(\begin{bmatrix}1\\2\end{bmatrix}\right) = \begin{bmatrix}2 & 0\\1 & 0\\4 & 3\end{bmatrix}\begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}2\\1\\10\end{bmatrix}$$

11) Find the inverse of the matrix below. (5 points)

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 4 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & 0 & 1 & -4 & 0 \\ 0 & 1 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 0 & 1 \\ 0 & 2 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -4 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$$