Name _____ Test 3, Spring 2021

1) Find all the eigenspaces of the matrix below (15 points)

$$\begin{bmatrix} -1 & -3 & 3 \\ 0 & -1 & 0 \\ 0 & -3 & 2 \end{bmatrix}$$

2) Given the basis below, find an orthogonal basis for the same vector space. (10 points)

$$\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}\right\}$$

3) Answer the following questions. (3 points each)

A) Let A be a 5×5 with eigenvalues $0, 0, 1, 2, 3$. What is the maximum rank of A?

B) Let A be a 3×5 matrix whose nullity is 4. When row reduced, how many rows of zeroes are there?

C) Consider a system of 4 equations and 4 variables that has a unique solution. When row reduced, how many pivots does the corresponding matrix have?

D) Let A be a 6×6 matrix whose corresponding linear transformation T is onto. Is T one-to-one?

E) Let A be a 3×3 matrix whose corresponding linear transformation T is not one-to-one. What is the determinant of A?

4) Given the two bases and linear transformation below, draw the diagram that represents this information. (10 points)

$$B_1 = \left\{\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}4\\2\end{bmatrix}\right\}, B_2 = \left\{\begin{bmatrix}1\\3\end{bmatrix}, \begin{bmatrix}0\\5\end{bmatrix}\right\}, T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}_{B_1}\right) = \begin{bmatrix}x_2\\3x_1 - x_2\end{bmatrix}_{B_2}$$

5) Continuing from the previous problem, find a formula for $[T^{-1}]_S^S$. (5 points)

6) Find the null space of the matrix below. (10 points)

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

7) Find the determinant of the matrix below. (10 points)

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 2 & 6 \\ 0 & 0 & 4 & 1 & 2 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

8) Find the characteristic polynomial of the matrix below. (5 points)

$$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$$

9) What matrix represents the quadratic form below?. (5 points)

$$x^2 + 4xy + 6y^2$$

For each of the following, answer true (always true) or false (any possible exception). No justification required, although if you choose to write a sentence explaining yourself you can receive partial credit for an incorrect response *if* you show some understanding of the underlying concepts. (5 points each)

T or F 10) It possible to multiply a matrix of size 3×5 with a matrix of size 6×3?

T or F 11) Let A be a 5×7 matrix whose rank is 5 and T the corresponding linear transformation given by $T(\vec{x}) = A\vec{x}$. T is onto.

T or F 12) Let A be a 5×5 matrix with eigenvalues $0, 1, 2, 3, 4$. A is diagonalizable.