Name ____ Test 3, Spring 2021

1) Find all the eigenspaces of the matrix below (15 points)

$$\begin{bmatrix} -1 & -3 & 3 \\ 0 & -1 & 0 \\ 0 & -3 & 2 \end{bmatrix}$$

$$|xI - A| = \begin{vmatrix} x+1 & 3 & -3 \\ 0 & x+1 & 0 \\ 0 & 3 & x-2 \end{vmatrix} = (x+1)\begin{vmatrix} x+1 & 0 \\ 3 & x-2 \end{vmatrix} = (x+1)(x+1)(x-2)$$

For $\lambda = -1$ we get:

$$\begin{bmatrix} -1+1 & 3 & -3 \\ 0 & -1+1 & 0 \\ 0 & 3 & -1-2 \end{bmatrix} = \begin{bmatrix} 0 & 3 & -3 \\ 0 & 0 & 0 \\ 0 & 3 & -3 \end{bmatrix} \sim_R \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has eigenspace $span\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right\}$.

For $\lambda = 2$ we get:

$$\begin{bmatrix} 2+1 & 3 & -3 \\ 0 & 2+1 & 0 \\ 0 & 3 & 2-2 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -3 \\ 0 & 3 & 0 \\ 0 & 3 & 0 \end{bmatrix} \sim_R \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

This has eigenspace $span\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right\}$.

2) Given the basis below, find an orthogonal basis for the same vector space. (10 points)

$$\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}2\\3\\4\end{bmatrix}\right\}$$

The first vector is $\begin{bmatrix}1\\1\\0\end{bmatrix}$

The second vector is $\begin{bmatrix}2\\3\\4\end{bmatrix} - proj_{\begin{bmatrix}1\\1\\0\end{bmatrix}}\left(\begin{bmatrix}2\\3\\4\end{bmatrix}\right)$

$$proj_{\begin{bmatrix}1\\1\\0\end{bmatrix}}\left(\begin{bmatrix}2\\3\\4\end{bmatrix}\right) = \frac{2+3}{2}\begin{bmatrix}1\\1\\0\end{bmatrix} = \begin{bmatrix}2.5\\2.5\\0\end{bmatrix}$$

$$\begin{bmatrix}2\\3\\4\end{bmatrix} - \begin{bmatrix}2.5\\2.5\\0\end{bmatrix} = \begin{bmatrix}-0.5\\0.5\\4\end{bmatrix}$$

The orthogonal basis is:

$$\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}-0.5\\0.5\\4\end{bmatrix}\right\}$$

3) Answer the following questions. (3 points each)

A) Let A be a 5×5 with eigenvalues $0, 0, 1, 2, 3$. What is the maximum rank of A?

4

B) Let A be a 3×5 matrix whose nullity is 4. When row reduced, how many rows of zeroes are there?

2

C) Consider a system of 4 equations and 4 variables that has a unique solution. When row reduced, how many pivots does the corresponding matrix have?

4

D) Let A be a 6×6 matrix whose corresponding linear transformation T is onto. Is T one-to-one?

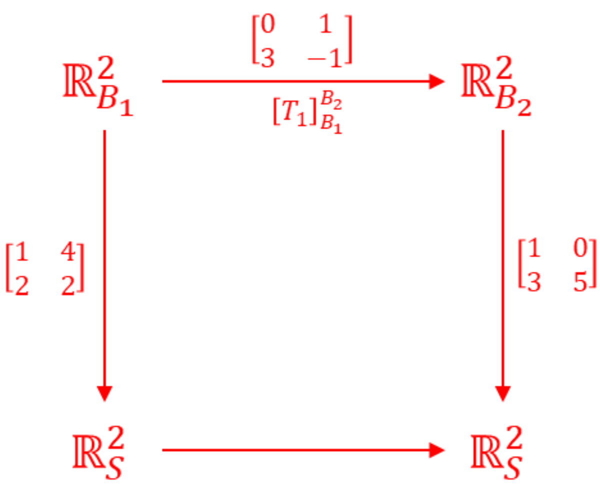
Yes

A) Let A be a 3×3 matrix whose corresponding linear transformation T is not one-to-one. What is the determinant of A?

0

4) Given the two bases and linear transformation below, draw the diagram that represents this information. (10 points)

$$B_1 = \left\{\begin{bmatrix}1\\2\end{bmatrix},\begin{bmatrix}4\\2\end{bmatrix}\right\}, B_2 = \left\{\begin{bmatrix}1\\3\end{bmatrix},\begin{bmatrix}0\\5\end{bmatrix}\right\}, T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}_{B_1}\right) = \begin{bmatrix}x_2\\3x_1 - x_2\end{bmatrix}_{B_2}$$



5) Continuing from the previous problem, find a formula for $[T^{-1}]^S_S$. (5 points)

$$\begin{bmatrix}1 & 4\\2 & 2\end{bmatrix}\begin{bmatrix}0 & 1\\3 & -1\end{bmatrix}^{-1}\begin{bmatrix}1 & 0\\3 & 5\end{bmatrix}^{-1}$$

6) Find the null space of the matrix below. (10 points)

$$\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$span\left\{\begin{bmatrix} -3 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}\right\}$$

7) Find the determinant of the matrix below. (10 points)

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 3 & 4 & 0 \\ 0 & 0 & 1 & 2 & 6 \\ 0 & 0 & 4 & 1 & 2 \\ 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

$$2 \cdot 1 \cdot 5 \cdot \begin{vmatrix} 1 & 2 \\ 4 & 1 \end{vmatrix} = 10(1-8) = -70$$

8) Find the characteristic polynomial of the matrix below. (5 points)

$$\begin{bmatrix} 2 & 3 \\ 4 & -1 \end{bmatrix}$$

$$\begin{vmatrix} x-2 & -3 \\ -4 & x+1 \end{vmatrix} = (x-2)(x+1) - 12$$

9) What matrix represents the quadratic form below?. (5 points)

$$x^2 + 4xy + 6y^2$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$$

For each of the following, answer true (always true) or false (any possible exception). No justification required, although if you choose to write a sentence explaining yourself you can receive partial credit for an incorrect response *if* you show some understanding of the underlying concepts. (5 points each)

T or F   10) It possible to multiply a matrix of size 3×5 with a matrix of size 6×3?

False

T or F   11) Let A be a 5×7 matrix whose rank is 5 and T the corresponding linear transformation given by $T(\vec{x}) = A\vec{x}$. T is onto.

True

T or F   12) Let A be a 5×5 matrix with eigenvalues $0, 1, 2, 3, 4$. A is diagonalizable.

True