Name __Solutions____ Discrete II, Quiz 1

Please do not simplify answers!

1) Seven guests arrive at Alice's house for a dinner party. However, Alice only has seating for three people (One by the window, one under the air conditioning vent, and the other in a chair that is missing a cushion). Being a very controlling host, Alice will select which guest will sit where. How many ways can Alice select the seating arrangement? (Alice herself will not sit, and is not one of the guests)

$$7 \cdot 6 \cdot 5$$

Note that this selection problem has:
Distinct objects because we can tell people apart.
No repetition because one person cannot sit in multiple chairs.
Ordered selection because we can tell the chairs apart.

2) At her job, Alice happens to be a password manager. She has decided that all company employees must abide by the following password regulations:

- Each password must be 8 or 9 characters.
- The first character must be a digit.
- The second, third, and fourth characters must be lowercase letters.
- The fifth character must be a 4.
- The sixth and seventh characters can be any lowercase digit, or a number.
- The eighth and [optionally] ninth characters must be one of the 6 symbols: !, @, #, $, %, ^.

How many different passwords are there?

$$10 \cdot 26 \cdot 26 \cdot 26 \cdot 1 \cdot 36 \cdot 36 \cdot 6 \cdot 7$$

OR

$$10 \cdot 26 \cdot 26 \cdot 26 \cdot 1 \cdot 36 \cdot 36 \cdot 6 + 10 \cdot 26 \cdot 26 \cdot 26 \cdot 1 \cdot 36 \cdot 36 \cdot 6 \cdot 6$$

The first 8 characters use the multiplication rule. For the 9^{th} character you either have you use the addition rule (you cannot have a password that is both 8 and 9 characters long), or you can think about the choices for the 9^{th} character as being one of the 6 given symbols, or the 7^{th} choice: missing.

3) How many integer solutions are there to:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 50$$

where $x_i \geq 0$ and $x_5 \leq 45$.

The answer to this problem is:
[Number of solutions with no additional restrictions on x_5] $-$ [Number of solutions with $x_5 \geq 46$]

$$\binom{50+7}{7} - \binom{4+7}{7}$$

OR

$$\binom{50+7}{50} - \binom{4+7}{4}$$

Number of solutions with no additional restrictions on x_5:
This is a "stars and bars" problem, there are 50 "1"s to distribute with 7 "+"s amongst them. That is, 57 objects with 7 of them chosen to be plusses. For example: the solution (6,8,1,16,0,12,5,2) would be:

111111+11111111+1+1111111111111111++111111111111+11111+11

Hence the number of solutions is the number of ways that we can take 57 distinguishable objects and choose 7 of them to be "+"s:

$$\binom{50+7}{7}$$

Number of solutions with $x_5 \geq 46$:
Here do a change of variables: $x_5' = x_5 - 46$. Then we get:

$$x_1 + x_2 + x_3 + x_4 + x_5' + x_6 + x_7 + x_8 = 4$$

Which is now another "stars and bars" problem: 4 "1"s with 7 "+"s. Hence the number of solutions is the number of ways that we can take 11 distinguishable* objects and choose 7 of them to be "+"s:

$$\binom{4+7}{7}$$

*They're considered distinguishable because you can tell the difference between making the 3rd object a "+" and making the 4th object a "+". However, the selections are indistinguishable/unordered in that if you choose the 3rd and 6th objects to be "+"s, that is the same as choosing the 6th and 3rd objects.