Name ______Solutions____ Discrete II, Quiz 3

1) Consider the expression $(x + 2y + 3z)^{34}$. Find the coefficient of $x^{10}y^{15}z^{9}$.

First note that:

$$(x + 2y + 3z)^{34} = \sum_{i+j+k=34} \binom{34}{i \quad j \quad k} x^i (2y)^j (3z)^k = \sum_{i+j+k=34} 2^j 3^k \binom{34}{i \quad j \quad k} x^i y^j z^k$$

Hence if we're interested in the $x^{10}y^{15}z^{9}$ term then:

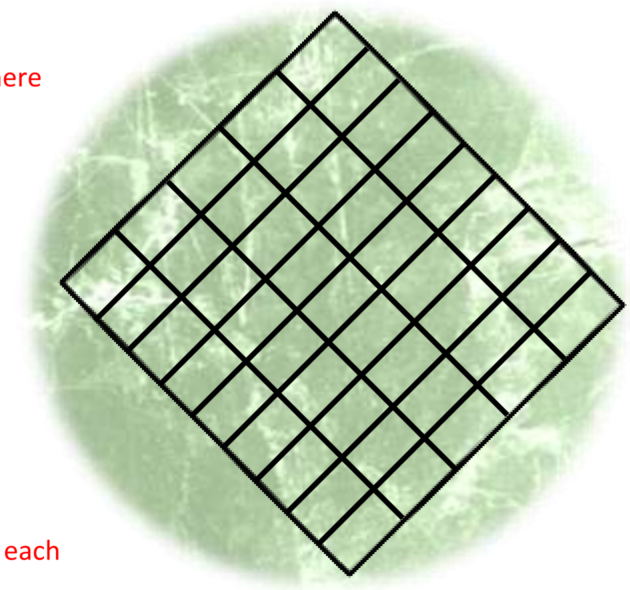
$$i = 10$$
$$j = 15$$
$$k = 9$$

This gives the coefficient as:

$$2^{15}3^{9}\binom{34}{10 \quad 15 \quad 9} = 2^{15}3^{9}\binom{34}{10}\binom{24}{15} = 2^{15}3^{9}\frac{34!}{10!\,24!}\frac{24!}{15!\,9!} = 110580908731068202352640$$

Best answer because it conveys the most information

2) The grid below depicts the possible routes that a drop of water may take while flowing down the face of a rock. (Yeah, it's oversimplified, but this drop of water is very particular about its pathing!). How many different routes are there for the drop of water to get from the top to the bottom?
If you express your answer using a recurrence relation, be sure to specify enough information so that the answer can be calculated from your answer.



The water can only go in downward directions, so it must ultimately go 5 units left (↙) and 9 units right (↘). Hence there are 14 units of travel, and we shall choose 5 of them to be the left units:

$$\binom{14}{5} = 2002$$

Or equivalently there are 14 units of travel, and we shall choose 9 of them to be right units:

$$\binom{14}{9} = 2002$$

Note that these are combinations because we do not care about order ($LLLRRLLRRRRRRR$ is the same as $LLLRRLLRRRRRRR$). That is, rearranging the lefts among each other does not change the route.

(The comment regarding recurrence relations ended up being obsolete; I was thinking of a slightly different problem where the water drop must stay on the left half of the rock.)