

Consider the grammar (N, T, P, σ) given as defined below.

$$N = \{ \langle s \rangle \}$$

$$T = \{ A, B, C, \dots, Z, \wedge, \vee, ! \}$$

$$P = \left\{ \begin{array}{l} \langle s \rangle \rightarrow \langle s \rangle \wedge \langle s \rangle \\ \langle s \rangle \rightarrow \langle s \rangle \vee \langle s \rangle \\ \langle s \rangle \rightarrow ! \langle s \rangle \\ \langle s \rangle \rightarrow A|B|C|\dots|Y|Z \end{array} \right\}$$

$$\sigma = \langle s \rangle$$

1a) How many valid words are in the language this grammar defines?

Infinitely many. For instance it contains:

$$\begin{array}{c} A \\ A \wedge A \\ A \wedge A \wedge A \\ A \wedge A \wedge A \wedge A \\ \vdots \end{array}$$

1b) Show that " $A \wedge B \vee !C$ " is in this language by providing a derivation.

$$\begin{array}{l} \langle s \rangle \Rightarrow \langle s \rangle \wedge \langle s \rangle \\ \Rightarrow \langle s \rangle \wedge \langle s \rangle \vee \langle s \rangle \\ \Rightarrow \langle s \rangle \wedge \langle s \rangle \vee ! \langle s \rangle \\ \Rightarrow A \wedge \langle s \rangle \vee ! \langle s \rangle \\ \Rightarrow A \wedge B \vee ! \langle s \rangle \\ \Rightarrow A \wedge B \vee ! C \end{array}$$

2) Write a grammar that generates the strings over $\{a, b, c\}$ that end in $abcba$.

A grammar is (N, T, P, σ) where:

$$N = \{x, y\}$$

$$T = \{a, b, c\}$$

$$P = \left\{ \begin{array}{l} y \rightarrow xabcba \\ x \rightarrow a|b|c|'''' \\ x \rightarrow xa|xb|xc \end{array} \right\}$$

$$\sigma = y$$

There are many correct answers.

If I wrote a derivation on your paper, it means your grammar has a valid word not ending in $abcba$.

If I wrote a string with a question mark, it means your grammar is missing that word.

