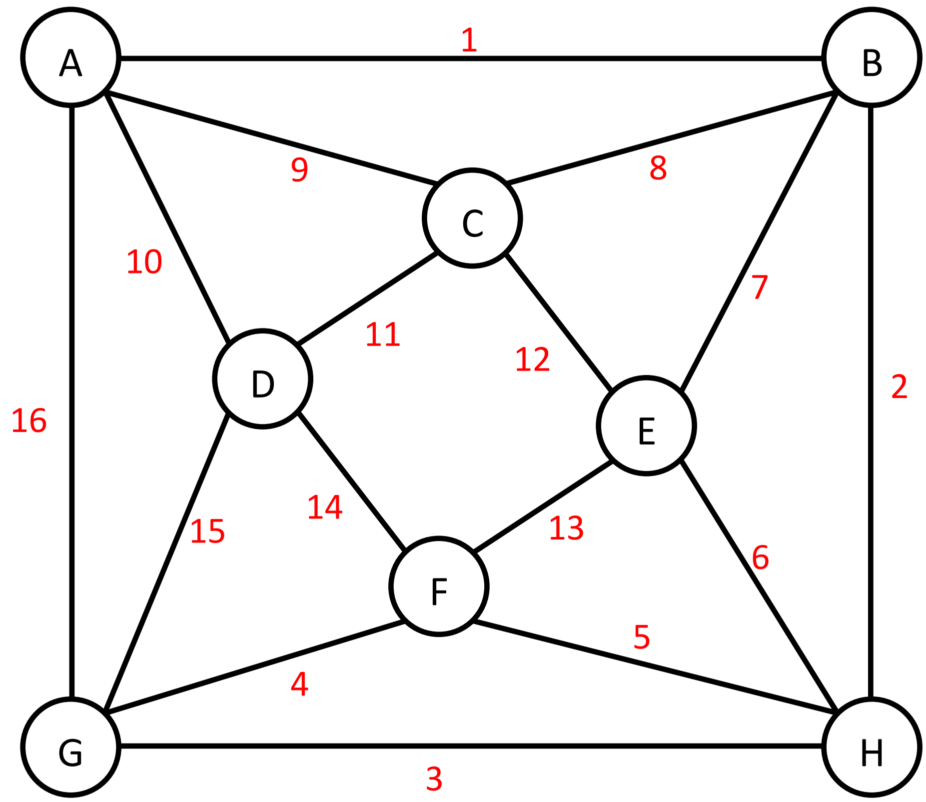
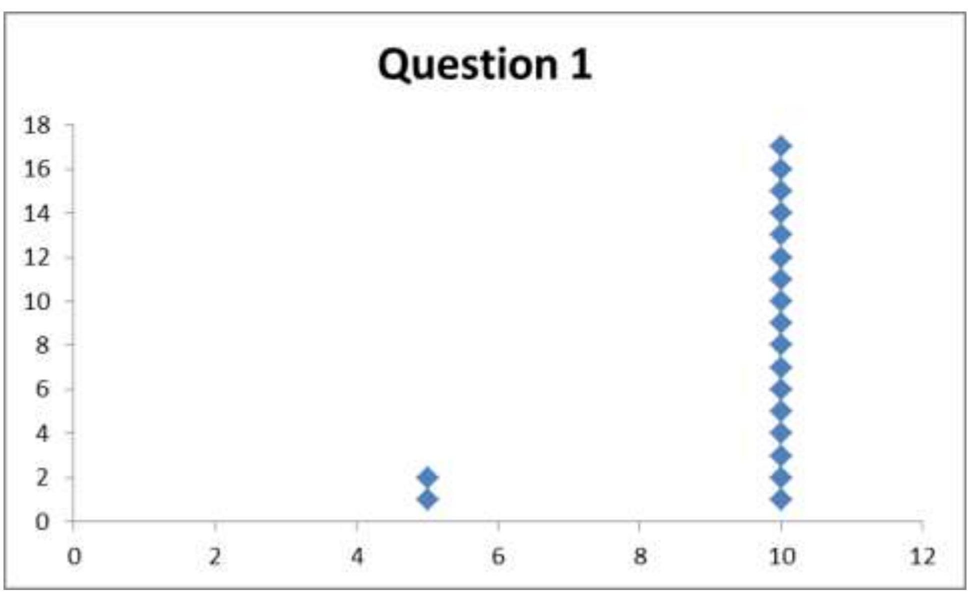
Name ___Solutions____ Discrete II, Test 2

1) Show that the graph below has a tour from vertex A through every edge exactly once, that then ends again at vertex A. (10 points)

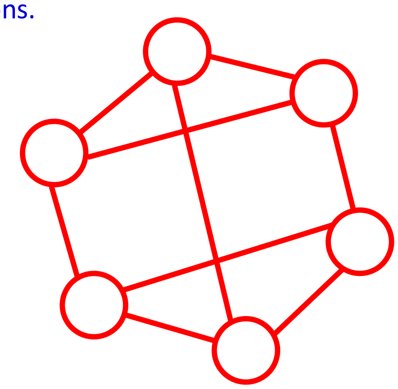


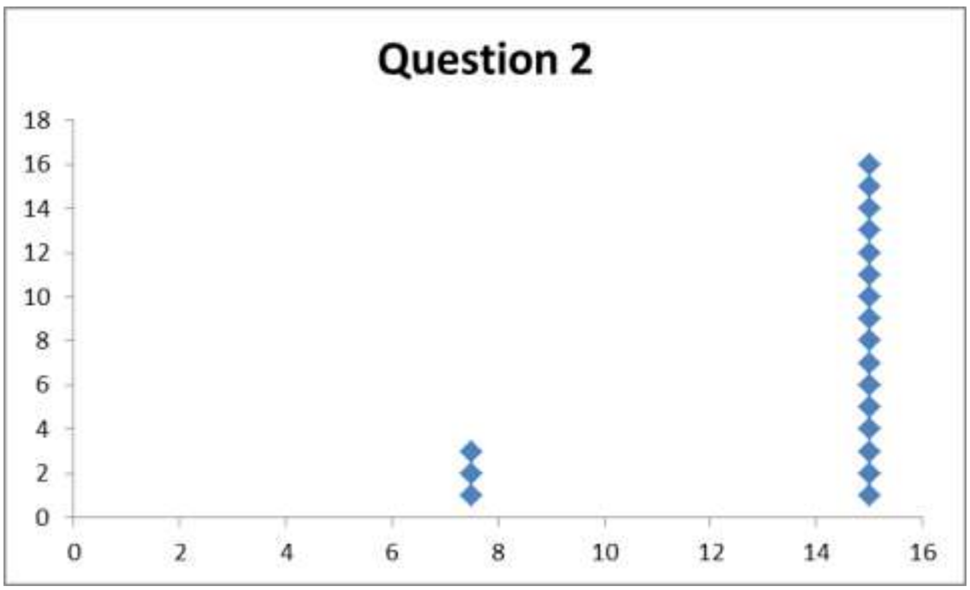
Either find such a tour (as above) or note that all the vertices have degree 4, which is even. Being a connected graph, it must have an Eulerian tour which is exactly what we're looking for.



2) Draw a graph having exactly 6 vertices, with each vertex of degree 3. (15 points)

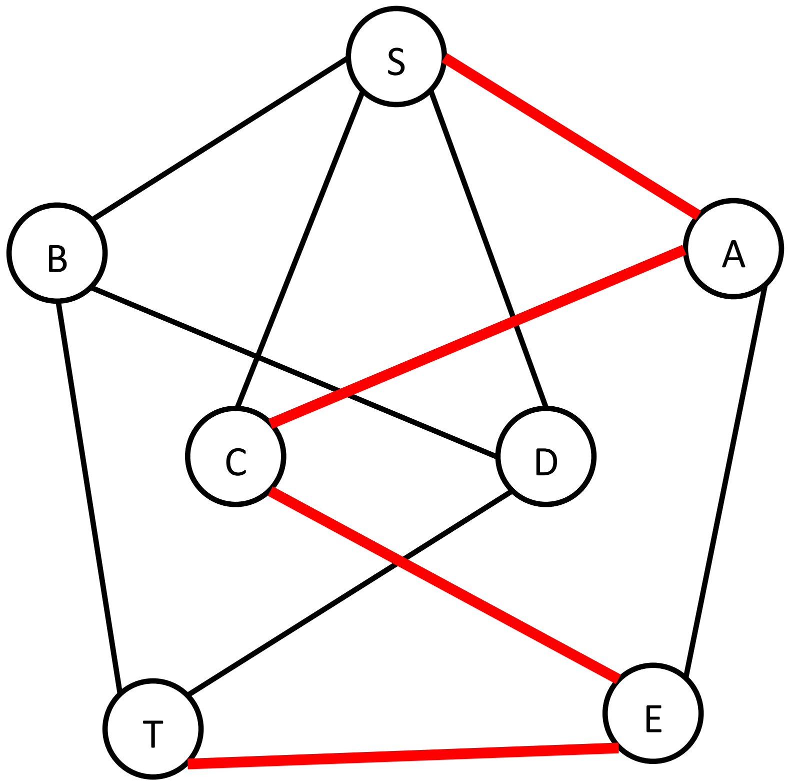
There are multiple solutions.

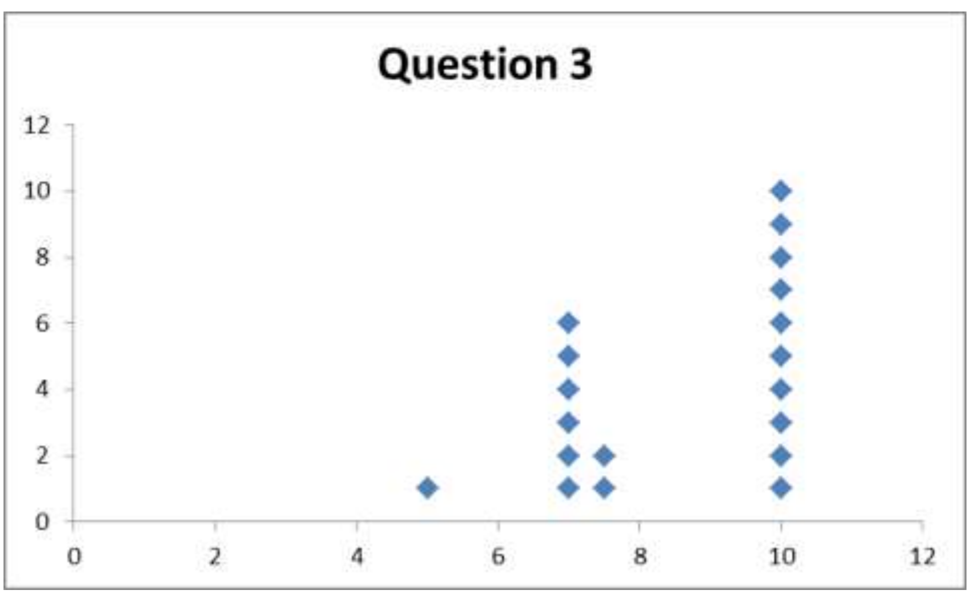




3) Use a depth-first search algorithm to find a path from S to T. Use the natural ordering on the alphabet. (10 points)

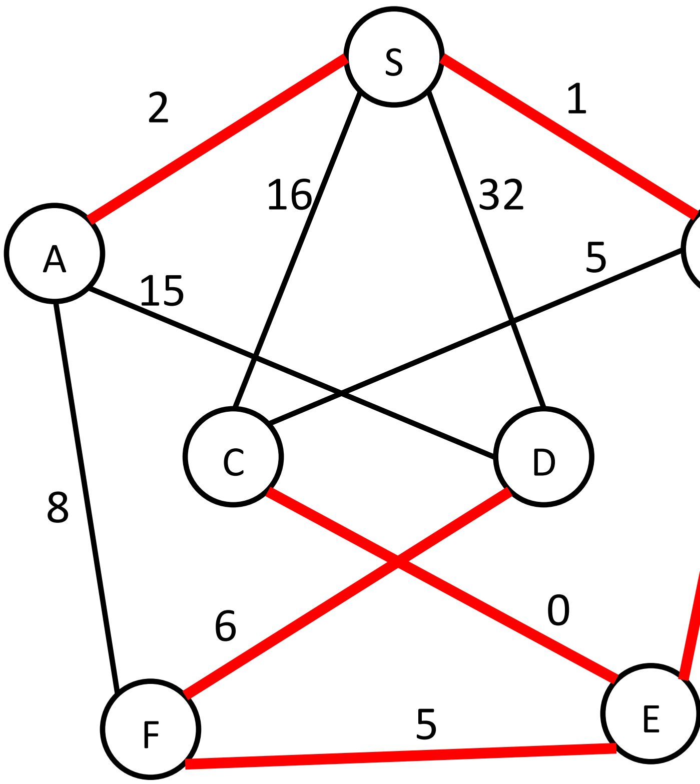
There were multiple accepted solutions: in class we focused on the concept, not a specific implementation of an algorithm.
This solution was obtained by using a priority queue and at each step inserting all new vertices as a block.

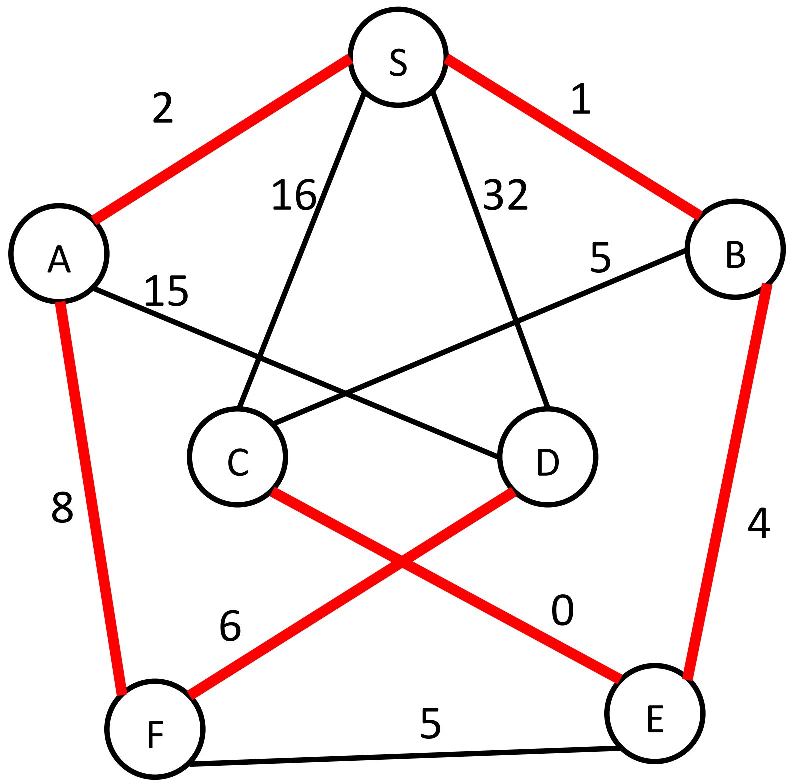


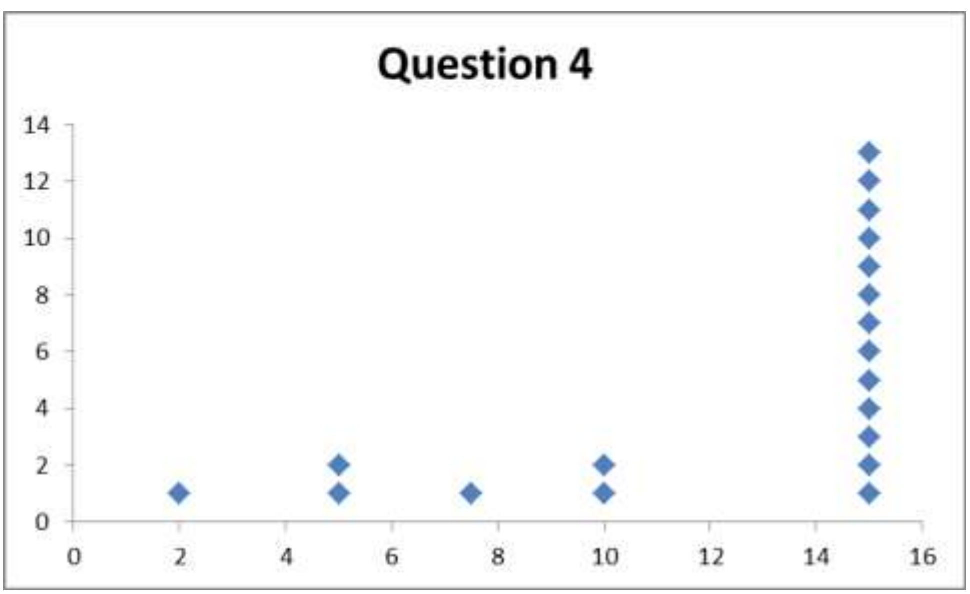


4) Use Dijkstra's Algorithm to find a shortest path tree rooted at vertex S, spanning the whole tree. (15 points)

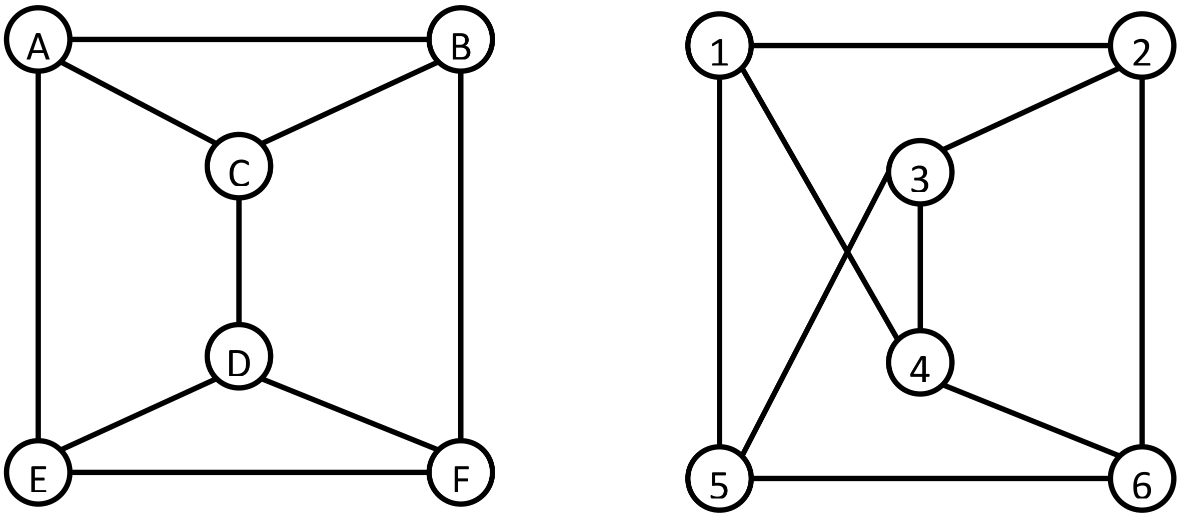
Either of the answers below are correct, depending on how you split a tie: take the new one or old one?



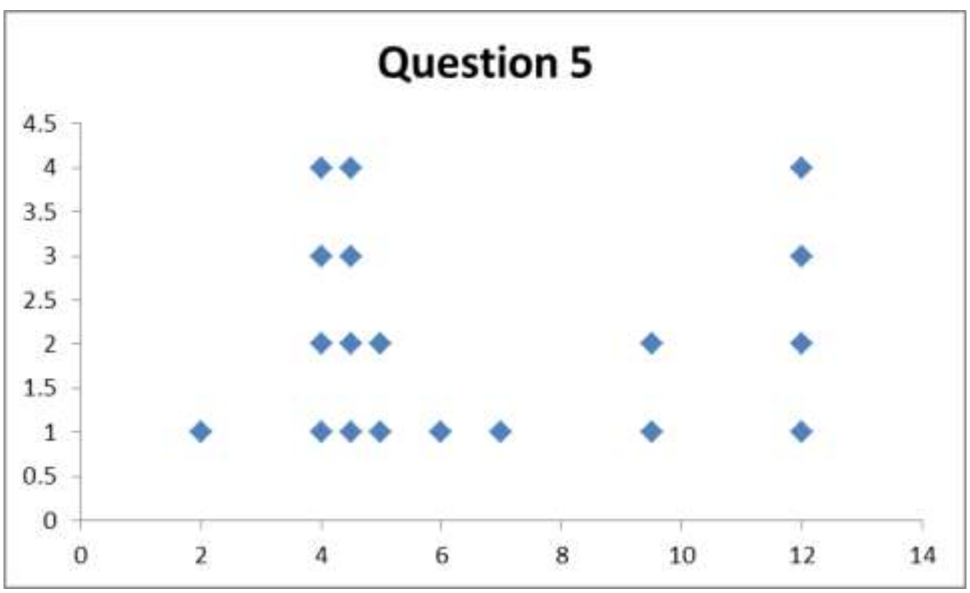




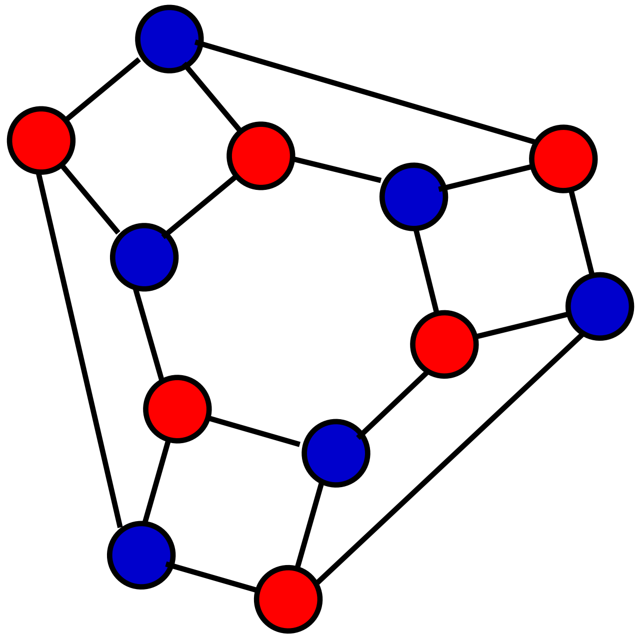
5) Determine whether the two graphs below are isomorphic. Justify your answer. (Answer: 2 points. Justification: 10 points)

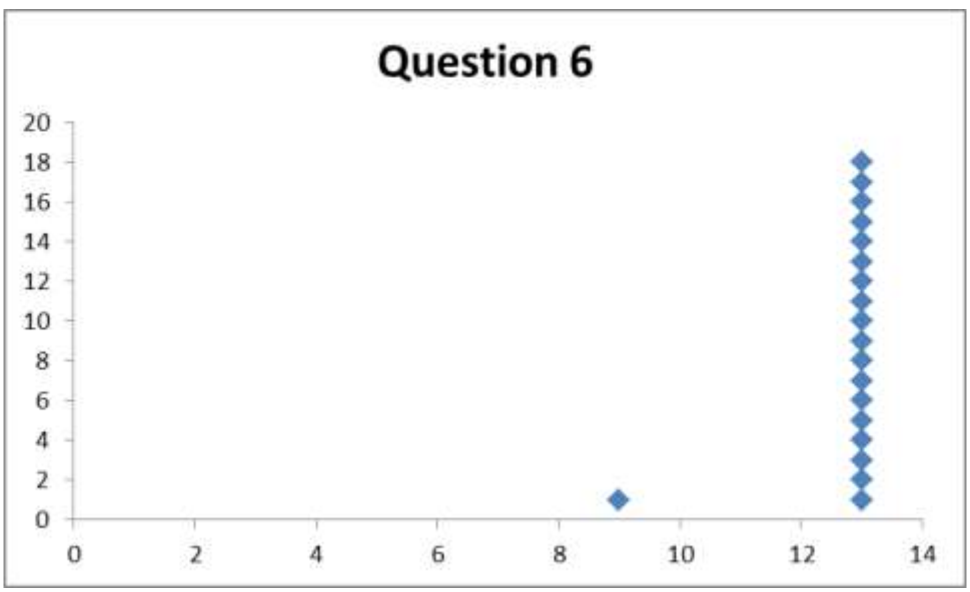


These are not isomorphic. Note in particular that the graph on the left has a C_3 subgraph, while the graph on the right does not. When justifying something, it is important to be precise. Several answers were vague enough that the particular graphs need not even matter!



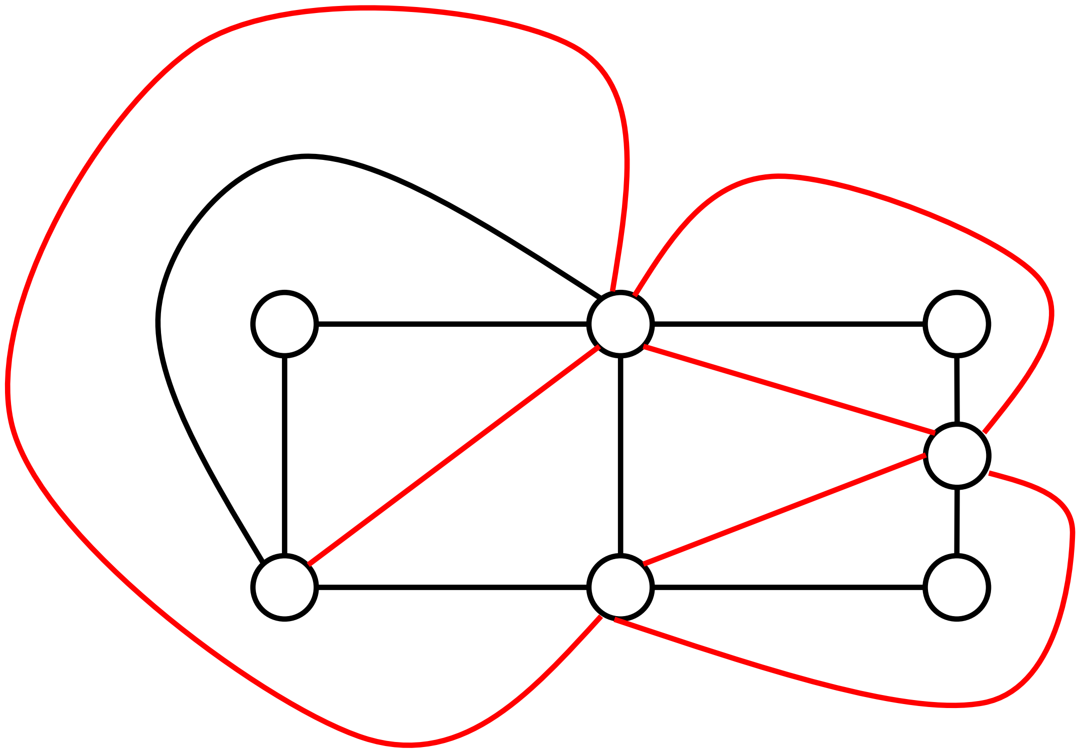
6) Color the vertices in the graph below using a proper coloring and as few colors as possible. (13 points)

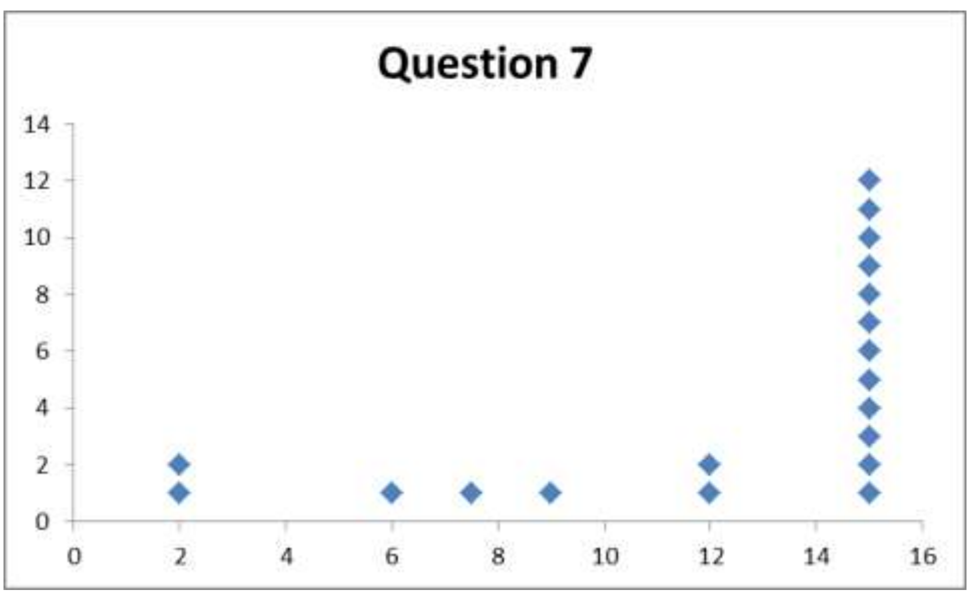




A triangulation of a planar graph is new graph obtained from the original by connecting as many vertices as possible while maintaining planarity (and of course never adding a parallel edge or loop). (15 points)
7) Find a triangulation of the following graph.

There are multiple solutions.



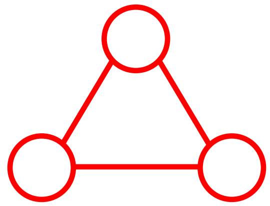


8) Explain why in a triangulation of a planar graph with at least 3 vertices: (10 points)

$$3f = 2e$$

Here f is the number of faces, and e is the number of edges.

The "3" is because every face is surrounded by three edges, as shown below.



The "2" is because when using the faces to count edges, each edge is counted twice.