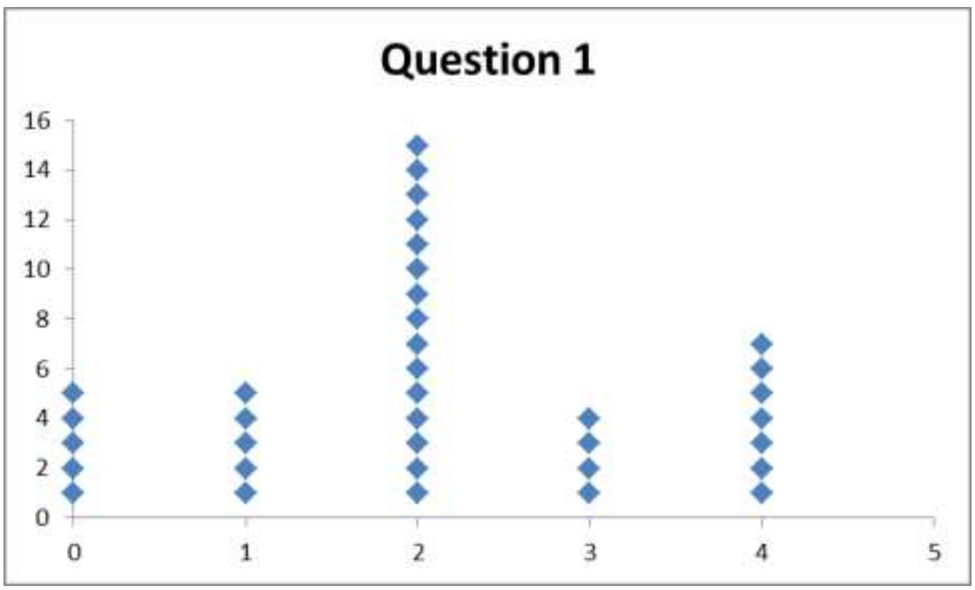
Name __Solutions____ Discrete II, Quiz 1, Spring 2016

Throughout this quiz, show all work and leave answers as meaningful expressions.

Use the following scenario for problems $1-2$.
Stephanie Agnail decides to save \$3,000 every year in an account with 2% interest each year. At the end of each year she first receives 2% interest on all money in the account, then she deposits another \$3,000. See the diagram on the board for an illustration of this scenario.
1) Write down the recurrence relation for the amount of money in the account after n years.

$$a_n = (1.02) \cdot a_{n-1} + 3000$$

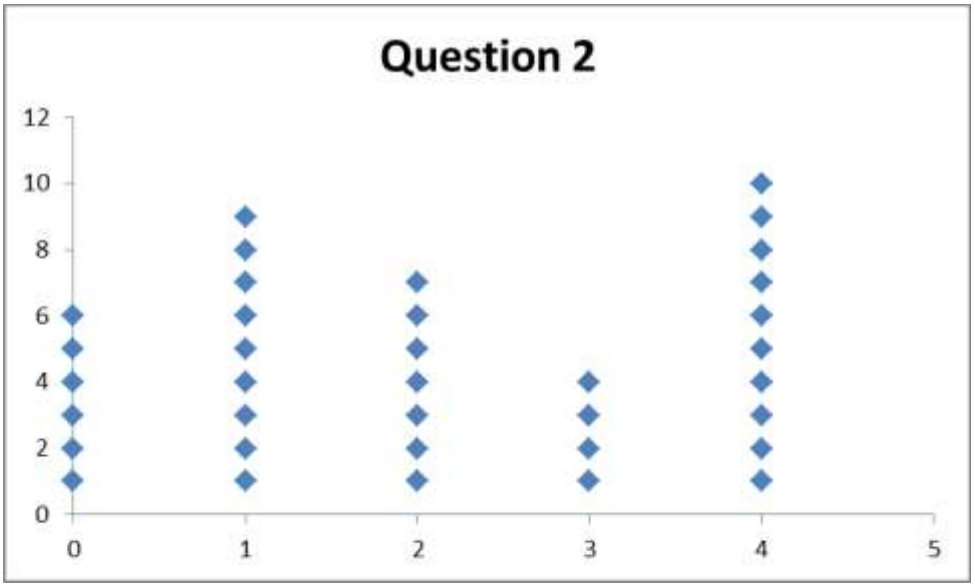
Be sure to write things that make sense. In particular, a_{n-1} is the sequence a at index $n-1$, but the expression "3000_{n-1}" makes absolutely no sense.



2) If the starting value in the account is $2,000, find the value of the account after 2 more years.

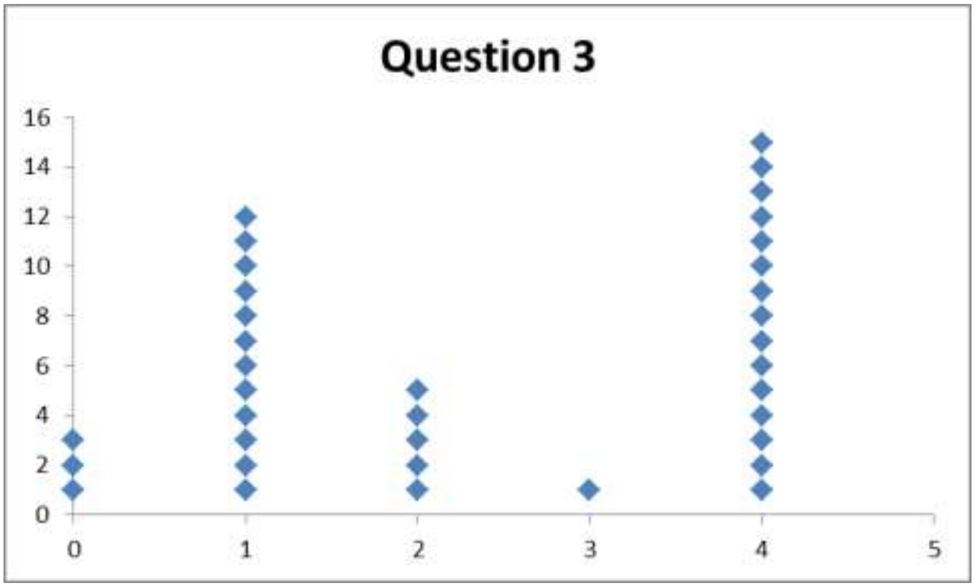
$$(2000 \cdot 1.02 + 3000) \cdot 1.02 + 3000$$

If you did the arithmetic to obtain 8140.8 be sure to do the work correctly. Numerous people either made mistakes or wrote complete gibberish.



3) How many 9-bit strings start with 0011?

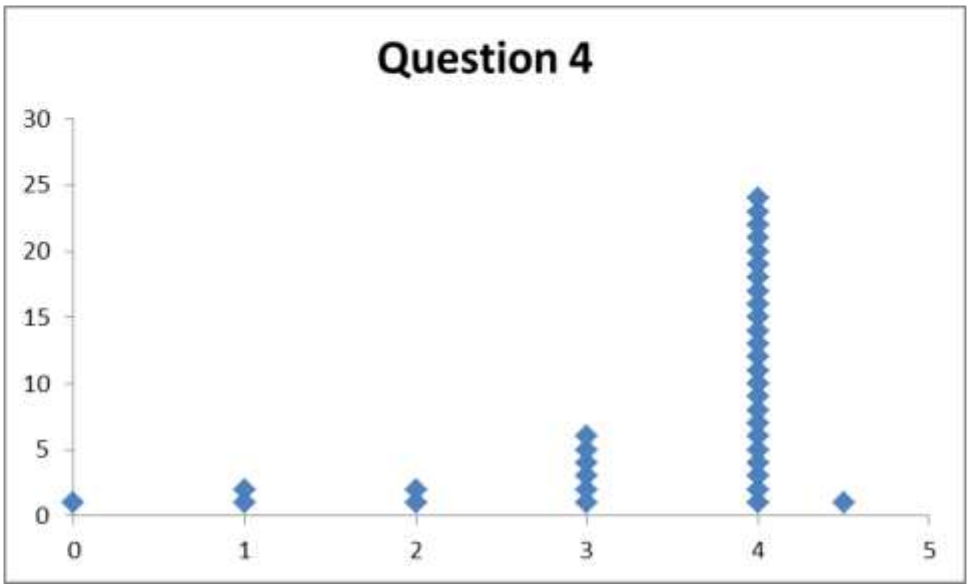
The first 4 bits are given in that there is only 1 way they can be 0011. The last 5 bits each have 2 options: 2^5



4) A restaurant has 7 appetizers, 14 main dishes, and 6 desserts. How many ways can you order 2 different appetizers, 1 main dish, and 2 desserts? You wouldn't mind getting the same dessert twice.

This is a multiplication problem because we're going to order the appetizers, main dish, and desserts.
Appetizers: $\binom{7}{2}$ --- 7 choices, choose two. Order doesn't matter.
Main dish: $\binom{14}{1}$ --- 14 choices, choose one.
Desserts: $\binom{2+5}{5}$ --- 2 choices to distribute between 6 bins.

$$\binom{7}{2}\binom{14}{1}\binom{7}{5}$$

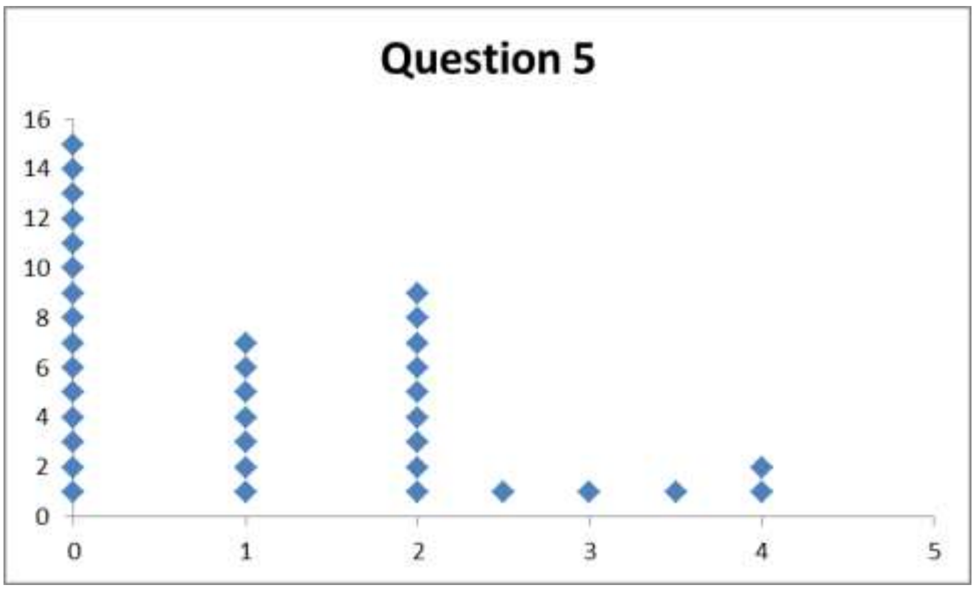


5) How many numbers between 1 and 600 are divisible by 2 or 3?

This is an inclusion-exclusion problem because we double count those divisible by both 2 and 3:
Divisible by 2: $\frac{600}{2}$ --- Every other number is divisible by 2.
Divisible by 3: $\frac{600}{3}$ --- Every third number is divisible by 3.
Divisible by 6: $\frac{600}{6}$ --- Above we counted numbers divisible by 6 in both categories. Every 6^{th} number is divisible by 6.

$$\frac{600}{2}+\frac{600}{3}-\frac{600}{6}$$

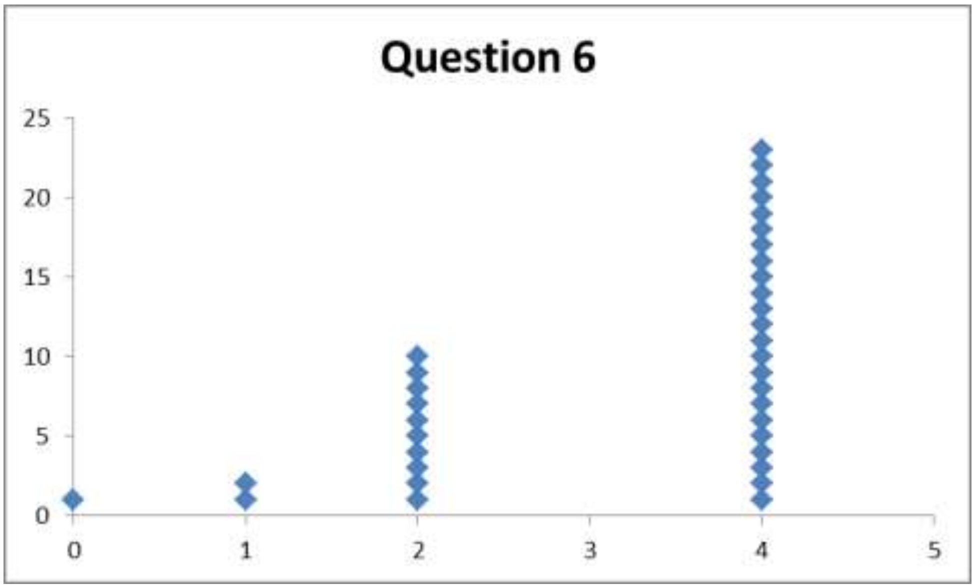
**Note: If 600 weren't divisible by both 2 and 3, you would have to be careful with details.



6) You roll a two dice. One is white, the other is red. How many different outcomes are there?

$$6^2 = 36$$

This was a **super easy** problem. If you didn't get full credit you've missed one of the fundamental concepts we've used every day for this entire chapter. I really really suggest you form a study group and go through a couple hundred counting problems before the test.



Consider the recurrence relation below for questions $7-9$.

$$a_n = 9a_{n-1} - 14a_{n-2}$$
$$a_0 = 1; a_1 = 17$$

7) Find a solution to the recurrence relation.

Set up the characteristic equation:

$$x^2 = 9x - 14$$

Solve the characteristic equation:

$$x^2 - 9x - 14 = 0$$
$$(x-2)(x-7) = 0$$
$$x = 2, 7$$

Plug into $a_n = x^n$ to get the solutions to the recurrence relation:

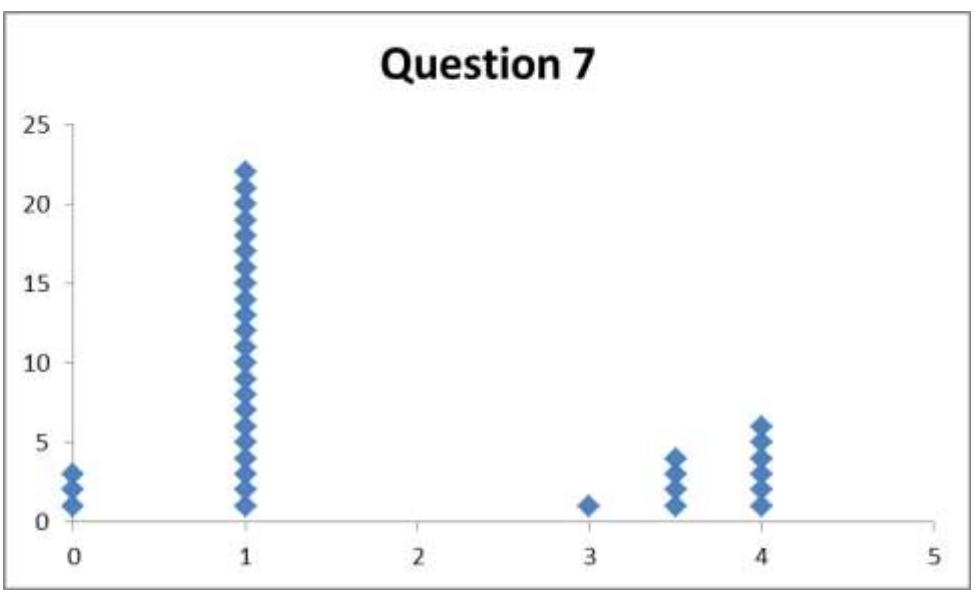
$$a_n = 2^n; a_n = 7^n$$

Choose one of them as your single solution to the recurrence relation:

$$a_n = 2^n$$

Grading comment: A lot of people that knew how to do the math missed most of the points on this problem. A solution to the recurrence relation and a solution to the characteristic equation are two very different things. You need to know what it is that you're doing and what the question is asking for.

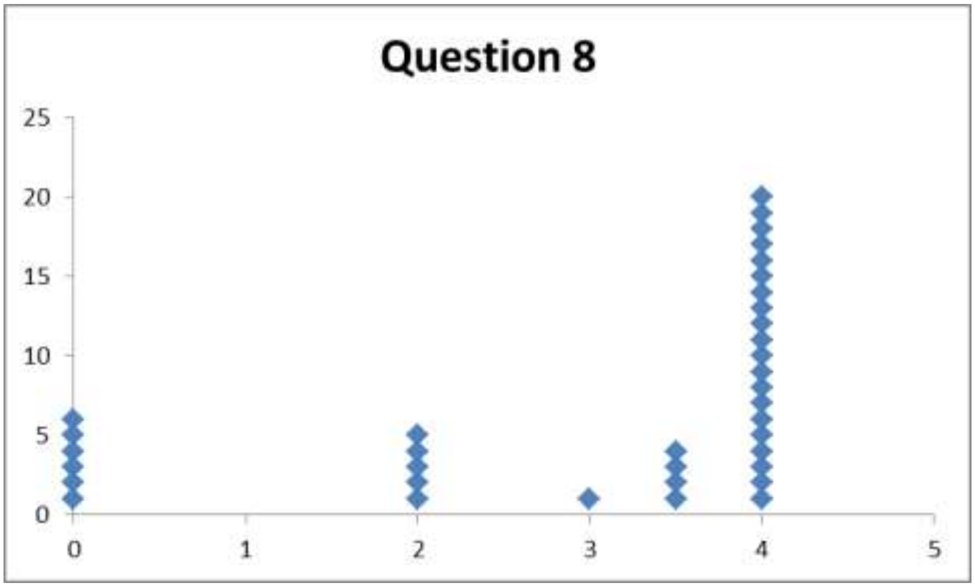
Also, circle your answer when there are so many things floating around the page.



8) Find the general solution to the recurrence relation.

The general solution is a linear combination of the solutions you found in the previous problem:

$$a_n = c \cdot 2^n + d \cdot 7^n$$



9) Find the particular solution to the recurrence relation with the given initial conditions.

Using the general solution, and initial conditions, set up two equations:

$$a_0 = 1 \Rightarrow 1 = c + d$$
$$a_1 = 17 \Rightarrow 17 = 2c + 7d$$

Solve using any method you like, I'll use elimination by subtracting twice the first equation from the 2nd:

$$15 = 5d$$
$$3 = d$$

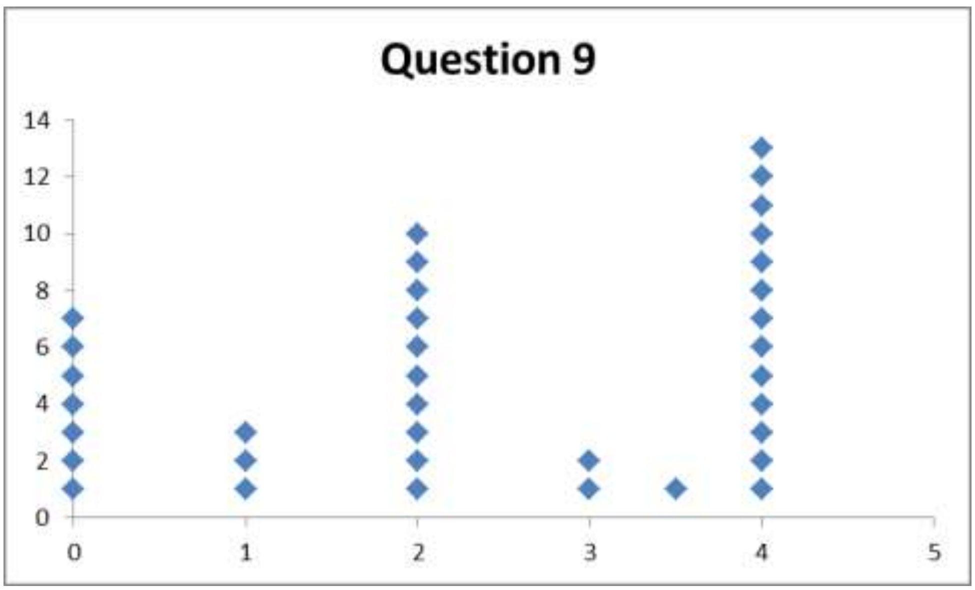
Plug back in to get the other variable:

$$1 = c + 3$$
$$c = -2$$

Plug both variables back in to the general solution:

$$a_n = -2 \cdot 2^n + 3 \cdot 7^n$$

Again, circle your answer!

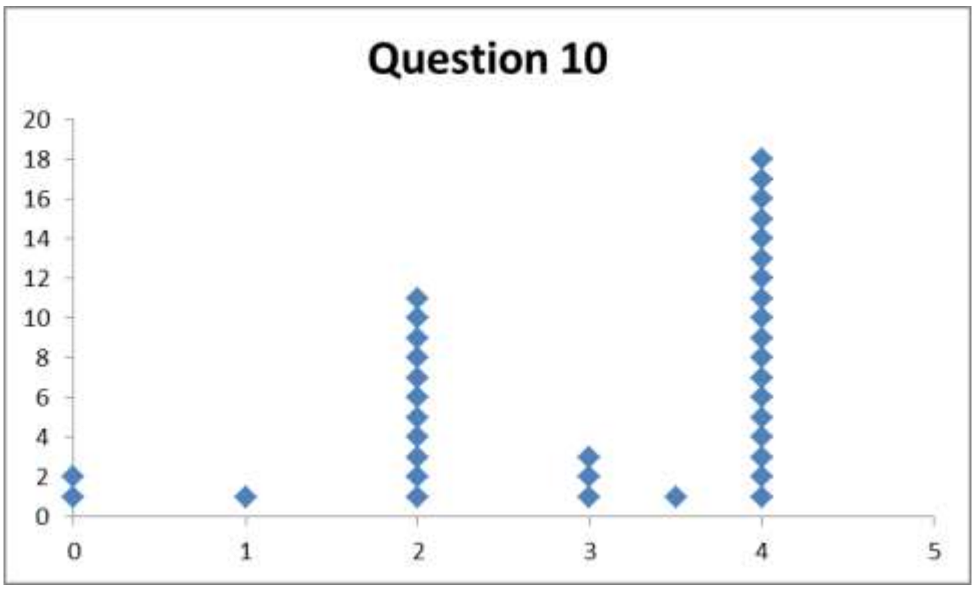


10) A book publishing company is shipping out the latest best-seller today. They are going to ship 1,000 copies divided in some manner between 7 retailers. How many ways can they divide the books between the retailers?

The books are identical, so which one goes to which retailer doesn't matter. We just need all 7 retailers to add up to 1000; e.g. we need to solve $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 1000$ for nonnegative integer solutions. Using the "stars and bars approach" we see that the answer is:

$$\binom{1000+6}{6} = \binom{1006}{6}$$

(Also, this is equal to $\binom{1006}{1000}$ if you select the stars instead of the bars)



11) Karl is giving out jelly beans. Every jelly bean is a different flavor, and Karl managed to get 33 different flavors! How many ways can he give 10 to you, 10 to Dr. Beyerl, and keep 13 for himself?

Give 10 to you: $\binom{33}{10}$

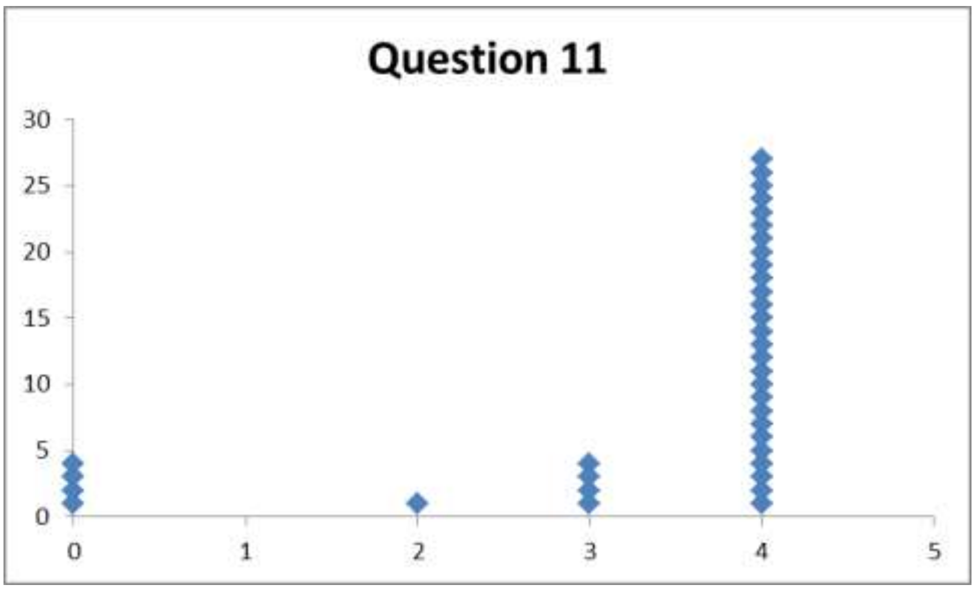
Now there are 23 left, give 10 to Dr. Beyerl: $\binom{23}{10}$

Now where are 13 left, give all 13 to himself: $\binom{13}{13} = 1$

He's doing all of these, so this is a multiplication problem:

$$\binom{33}{10} \cdot \binom{23}{10} \cdot \binom{13}{13}$$

(Also, this is equal to $\binom{33}{10\ 10\ 13}$ if you used a multicombination)



12) Calculate $\binom{4}{3}$. (Do the arithmetic until you get a single number as your answer)