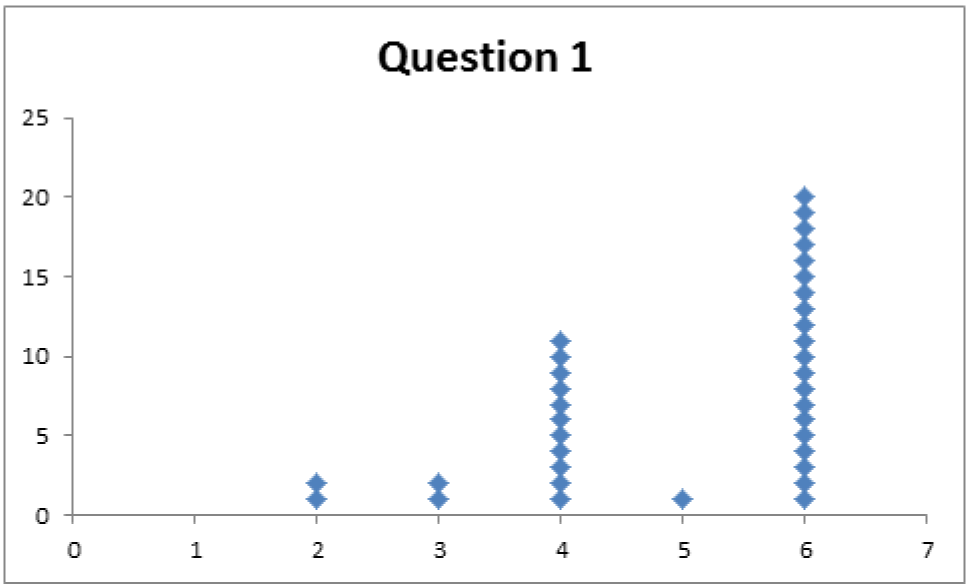
Name ____ Discrete II, 2/24/2016, Quiz 2

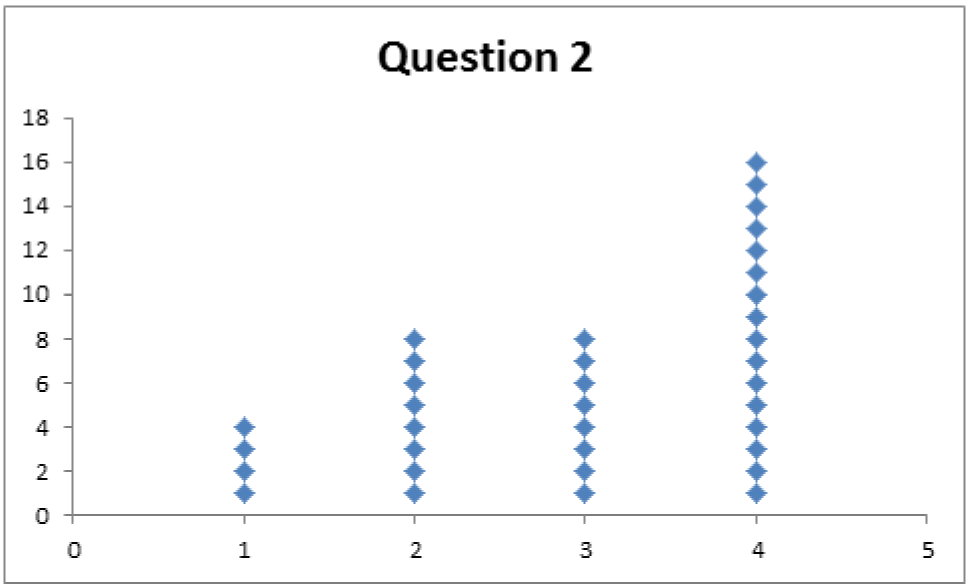
1) List all 2-combinations from the set $\{a, b, c, d\}$ in lexicographic order. (6 points)

ab
ac
ad
bc
bd
cd



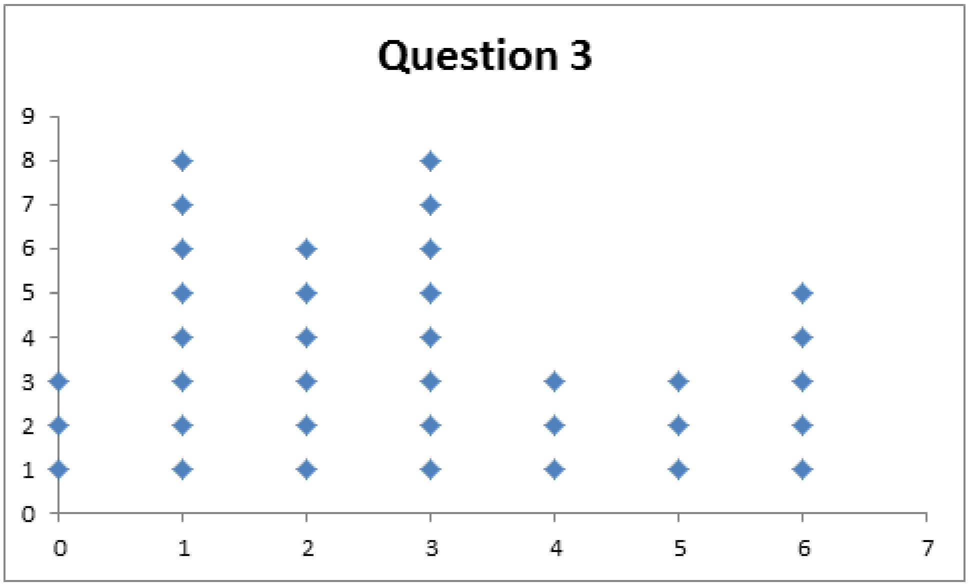
2) Suppose you are enumerating all permutations from the set $\{1, 2, 3, 4, 5\}$ and you are currently at "12534″. Find the next 4 permutations in lexicographic order. (4 points)

12543
13245
13254
13425



3) You roll 7 dice. What is the probability of getting two 3's and five 4's? (6 points)

$$\frac{\binom{7}{2}\binom{5}{5}}{6^7}$$

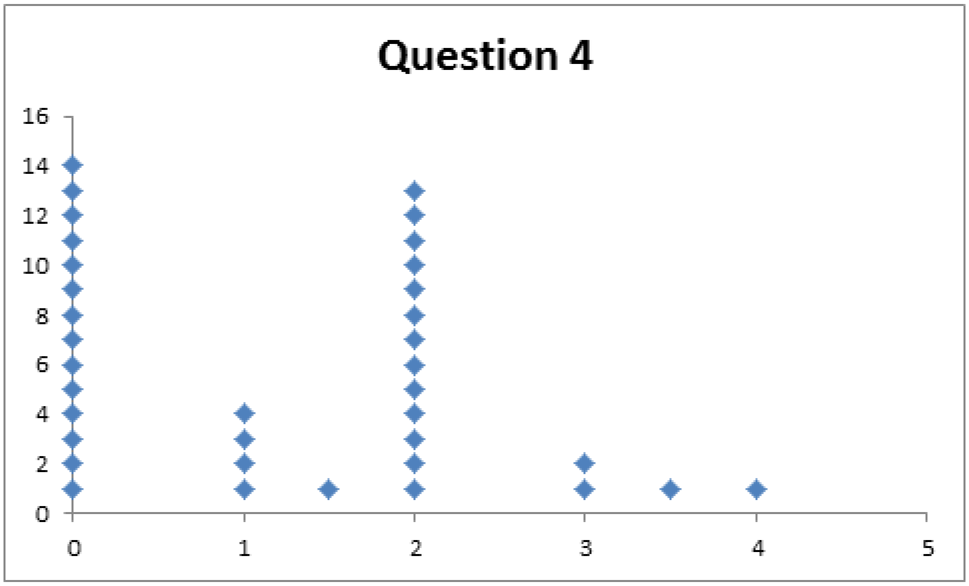


4) What is the coefficient of x^8 in $(3+2x)^{17}$? (4 points)

$$\binom{17}{8}3^{17-8}(2x)^8 = \binom{17}{8}3^9 2^8 x^8$$

Hence the coefficient is:

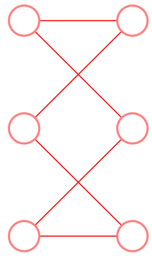
$$\binom{17}{8}3^9 2^8$$

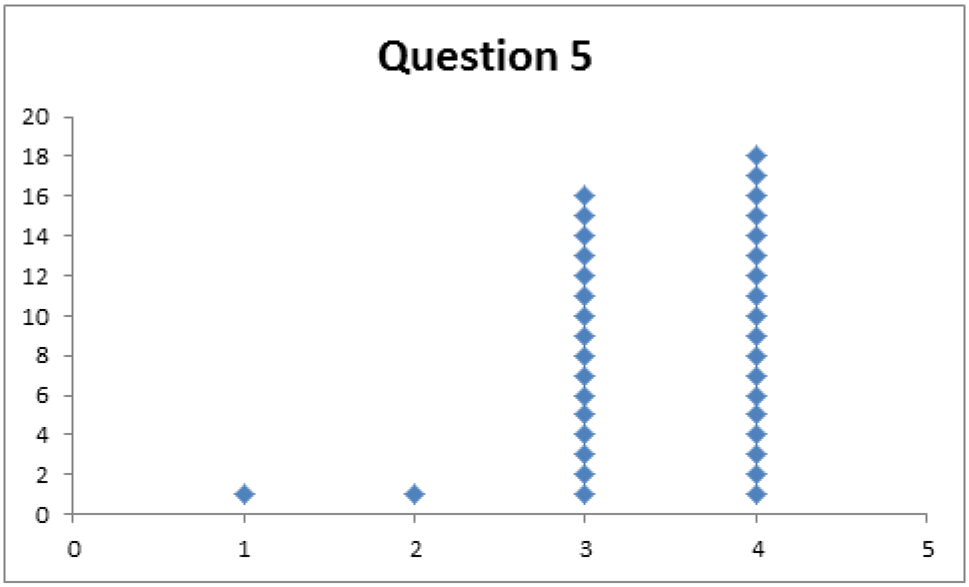


5) Draw an example of a bipartite graph with 6 vertices such that every vertex has degree 2.

(4 points)

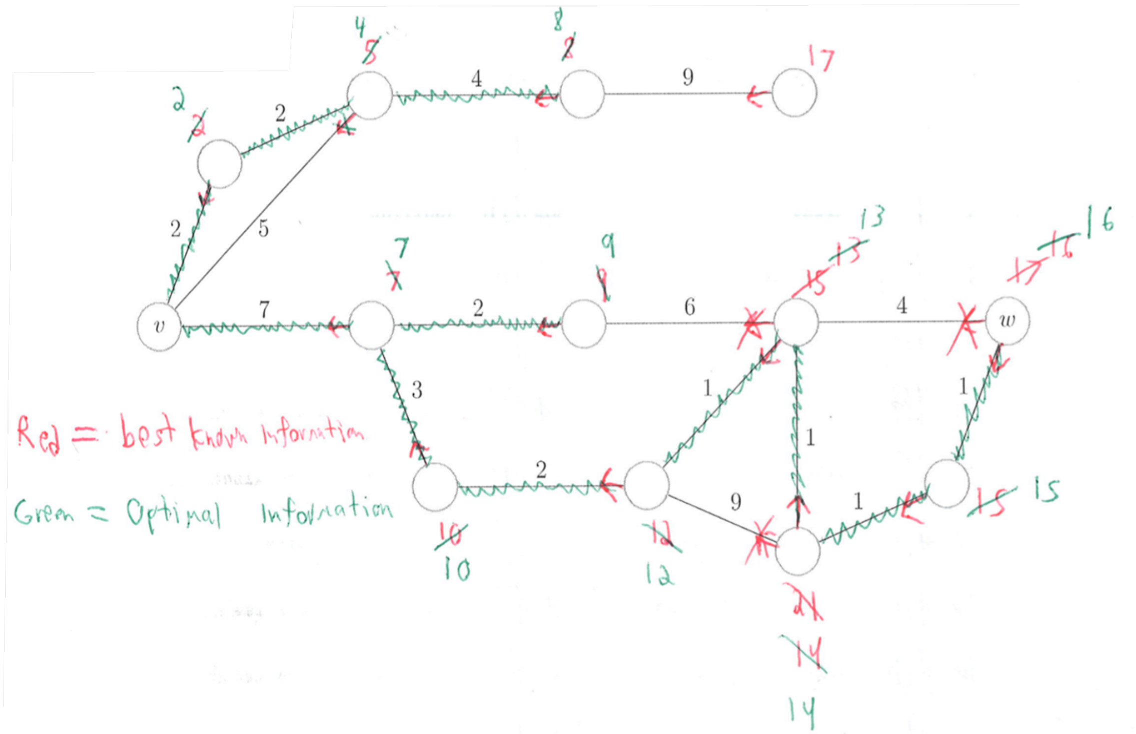
There are several examples, this is one of them:





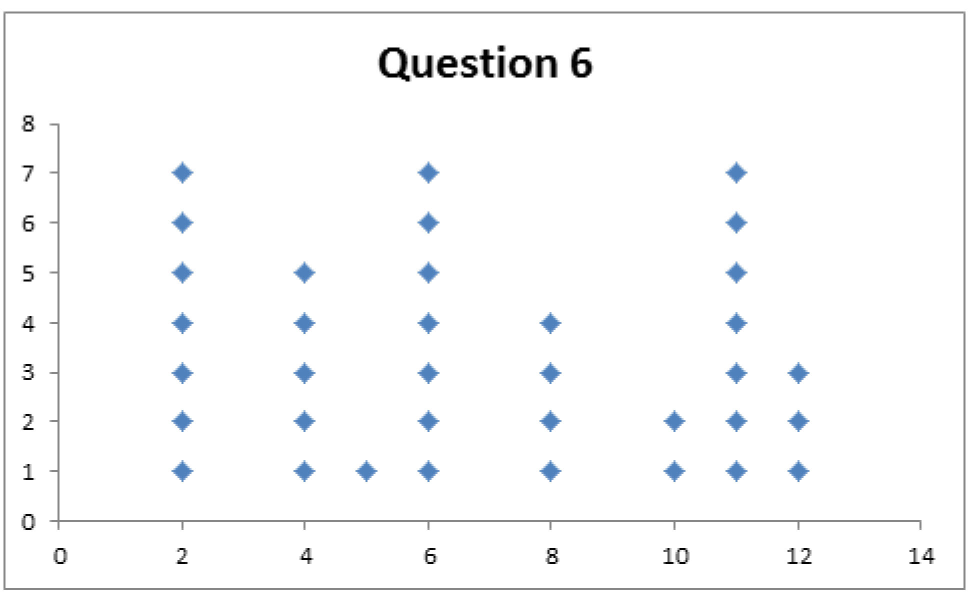
6) On the graph below, use Dijsktra's algorithm to find the shortest path from v to w. Please use different colors to signify different meanings, and make a legend below to explain the colors. (12 points)

As can be seen below, the shortest path has a length of 16, passing through the green edges.

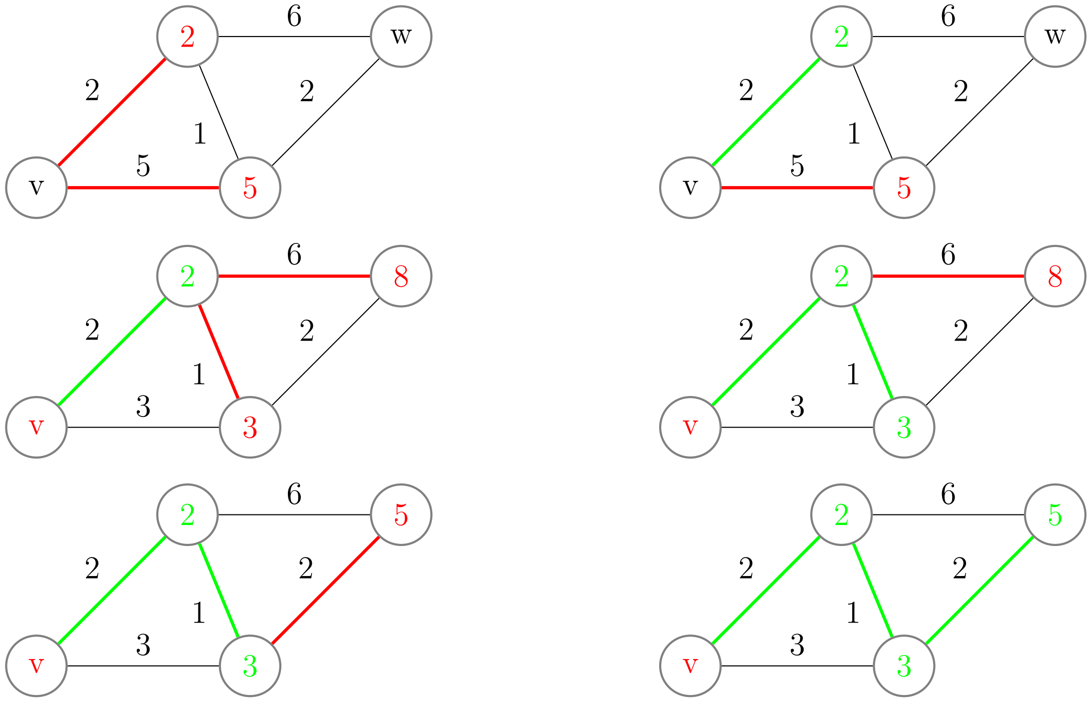


The scores on this problem were quite low. Every mark on the diagram above is important. Here are some of the common mistakes I saw:

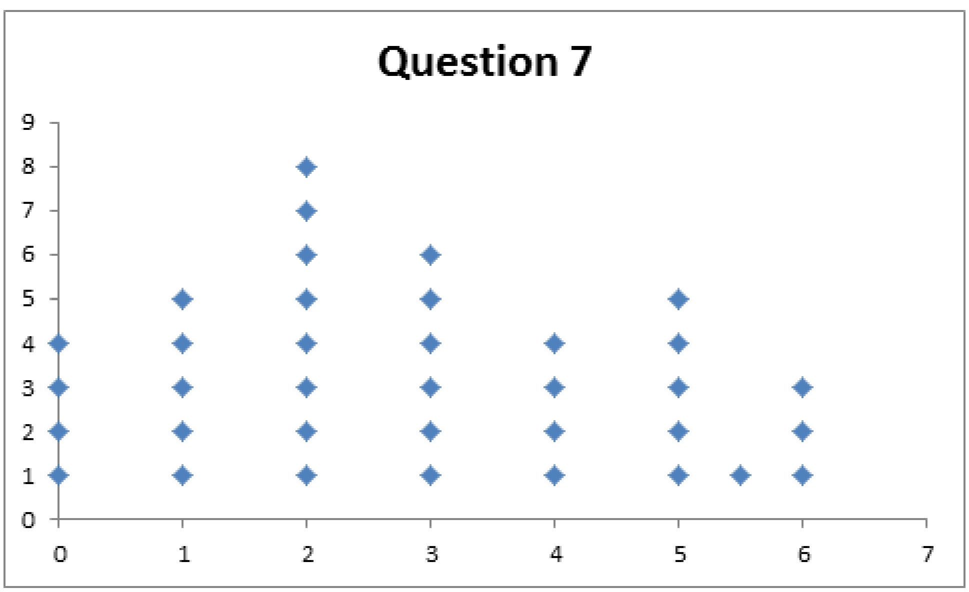
- Not running Dijsktra's algorithm. The point is not to find the shortest path, that's easy enough on a small graph. The point is to show that you know how to run the algorithm we learned because it is an algorithm that will scale to graphs that have millions of vertices - or more.
- Applying some of the ideas behind Dijsktra's algorithm, but ignoring parts of the graph because you know they aren't optimal. Again, you need to know how the algorithm works, and it can't see the entire graph from the start.
- Applying some parts of the algorithm, but skipping other parts such as labeling optimal routes, optimal distances, and best-known distances. This includes, for instance, best-known distances that are later updated.



7) Below is a graph (actually 6 drawings of it). You can run Dijkstra's algorithm on this graph in at most 6 steps - do so, using each new graph to illustrate one new step. (6 points)

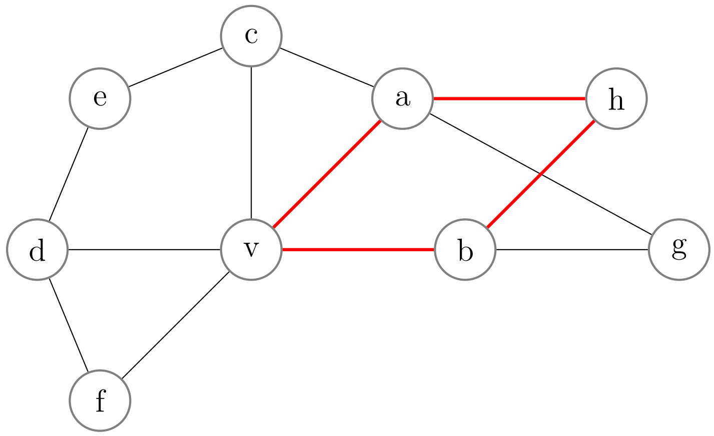


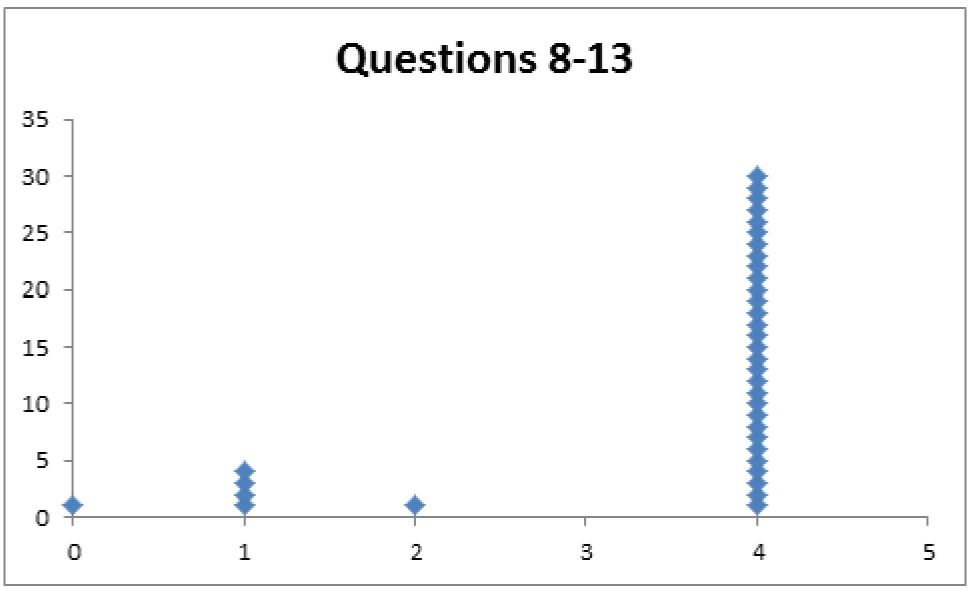
Here red represents best-known routes and distances, while green represents optimal routes and distances. Each pair of graphs is actually one iteration in Dijskstra'a algorithm, but I separated it into two parts: first finding all new best-known routes, then a new graph for adding the best new vertex with its optimal route.



8) In the graph below, identify a cycle of length 4 through vertex v that does not include vertex d. (4 points)

There are a couple such cycles. Below is one of them.





9) Sketch a graph with the adjacency matrix given below. (4 points)

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Here we'll label the five vertices v_1, v_2, v_3, v_4, and v_5 corresponding to the 5 rows and columns in the matrix. We then have: