1) A planar graph has 6 vertices and 8 edges. How many faces does it have? (4 points)

2) Sketch a graph of a planar graph has 6 vertices and 8 edges. Be sure your drawing is planar. (4 points)

3) A complete binary tree graph has 3 levels. How many vertices does it have? (4 points)

4) Sketch a graph of a complete binary tree with 3 levels. Be sure your drawing makes it obvious it's a complete binary tree. (4 points)

5) A "3-regular graph" is defined as graph in which each vertex has degree exactly 3. Additionally, we'll label each of the n vertices with an integer label, and store the edges in a $3 \times n$ matrix. What is the asymptotic runtime of a properly implemented depth-first search in this situation? (6 points)

6) Find a spanning tree of the graph below, using a breadth-first construction. Label the vertices in the order in which you add them to the tree. Start from the vertex labeled 'a' and evaluate vertices in lexicographic order. (6 points)
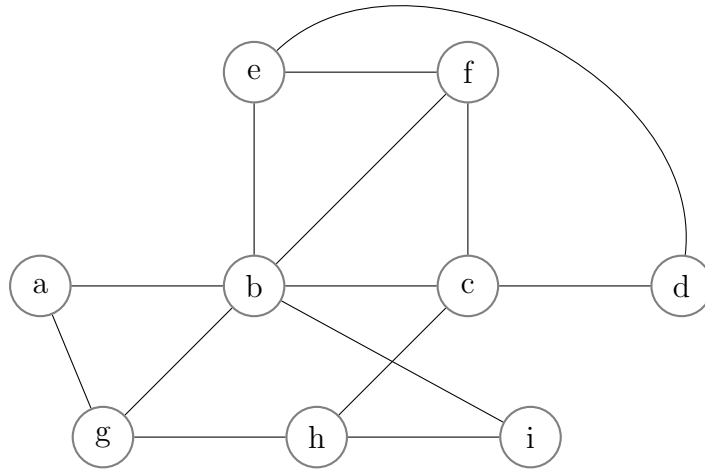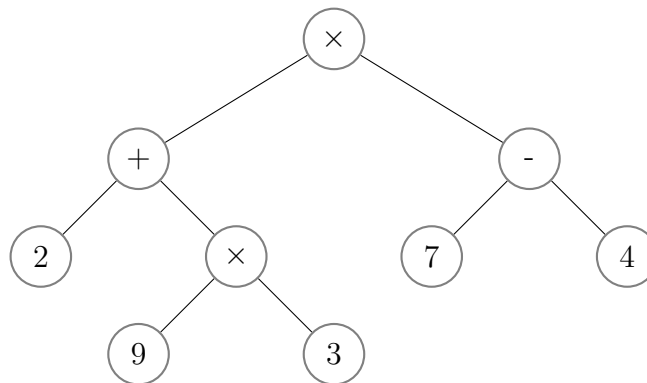

7) Find a minimal spanning tree on the graph shown below. (4 points)
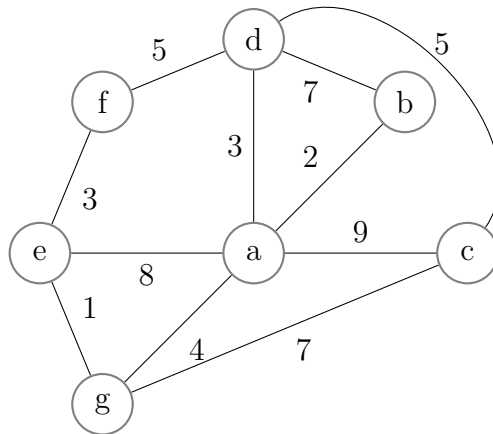
8) A tree has m leaves. Find a bound on the total number of vertices. (4 points)

9) Construct the expression tree for the mathematical expression $2 + 3 \cdot 4$. (6 points)

10) Write the postfix expression corresponding to the expression tree below. Do not evaluate it. (4 points)

11) In a StarCraft tournament, there are n contestants. The contestants, in pairs, play one game against each other and there is always a winner. Find and justify a bound on the total number of games required to determine the best player. Assume that skill is transitive: if a defeats b and b defeats c, then it is assumed that a could defeat c. (4 points)

12) Choose one: Sketch all binary trees with 3 vertices OR how many binary trees are there on 5 vertices? (0 points)

13) Sketch a transition diagram for a finite state machine with 2 states S and T, 2 input values a and b, and 2 output values x and y. (0 points)

14) Sketch a transition diagram for a finite state automata that accepts all strings over the alphabet $\{a, b\}$ that end in b. (0 points)

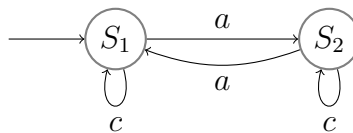15) Give a derivation to show that $abbc$ is in the grammar $G = \{N, T, P, \sigma\}$ defined below. (0 points)

$N = \{S, U\}$
$T = \{a, b, c\}$
$P = \{S \to a|b|c|U, U \to aUU|bUU|cUU|c, UU \to c, abUUU \to b\}$
$\sigma = S$

16) Given the deterministic finite state automata below, find the corresponding nondeterministic finite state automata. (0 points)

17) Given the nondeterministic finite state automata below, find the corresponding deterministic finite state automata. (0 points)