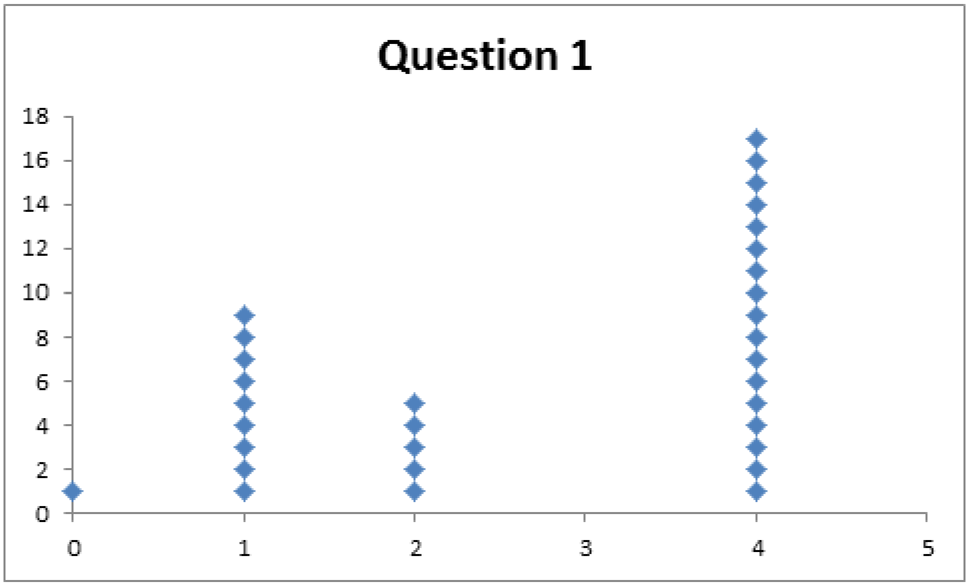
Name ________________________________ Discrete II, 4/8/2016, Quiz 3

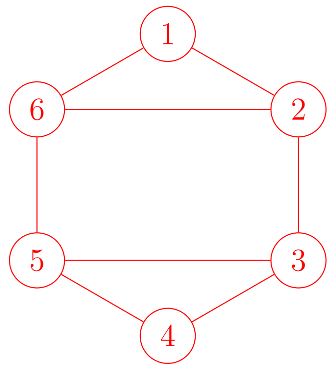
1) A planar graph has 6 vertices and 8 edges. How many faces does it have? (4 points)

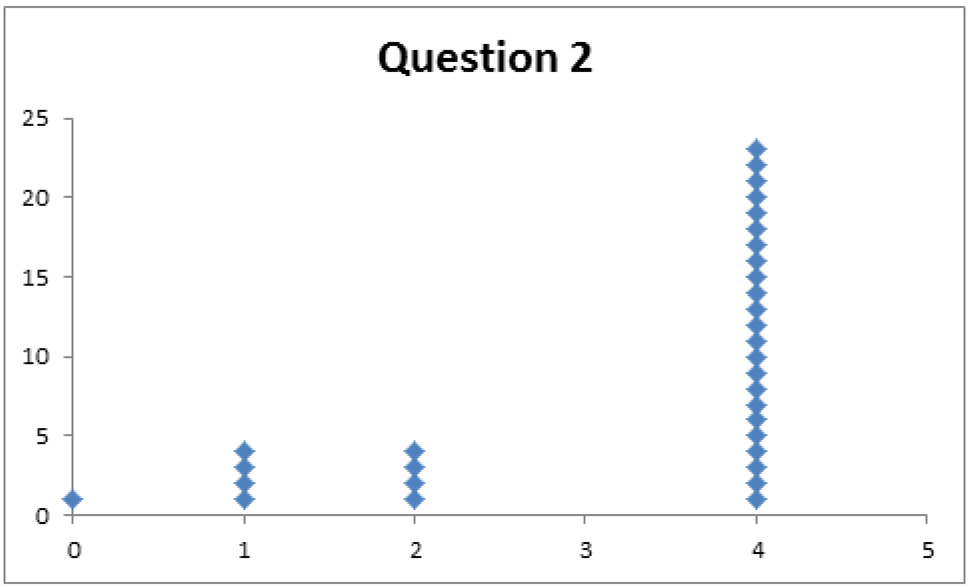
F=E-V+2=8-6+2=4



2) Sketch a graph of a planar graph has 6 vertices and 8 edges. Be sure your drawing is planar. (4 points)

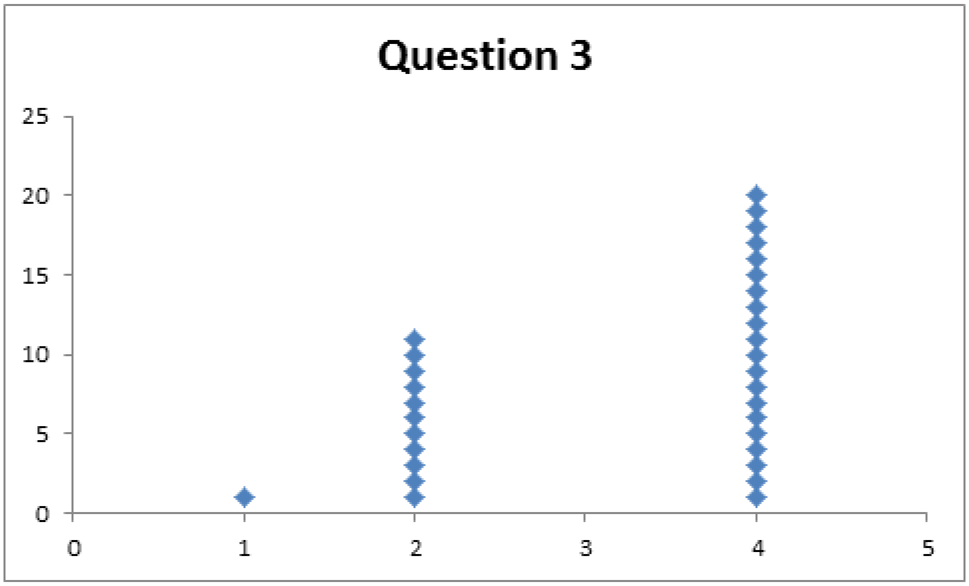
There are many solutions. Below is one example.



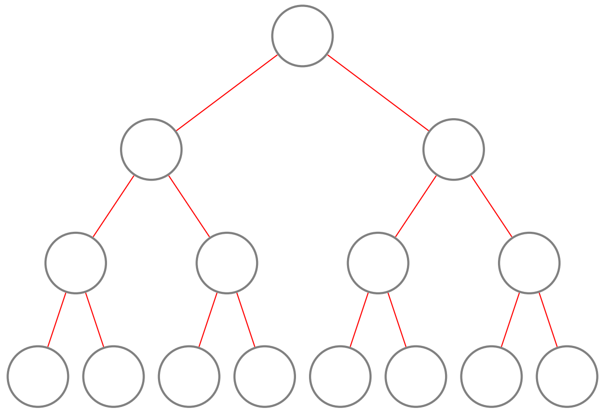


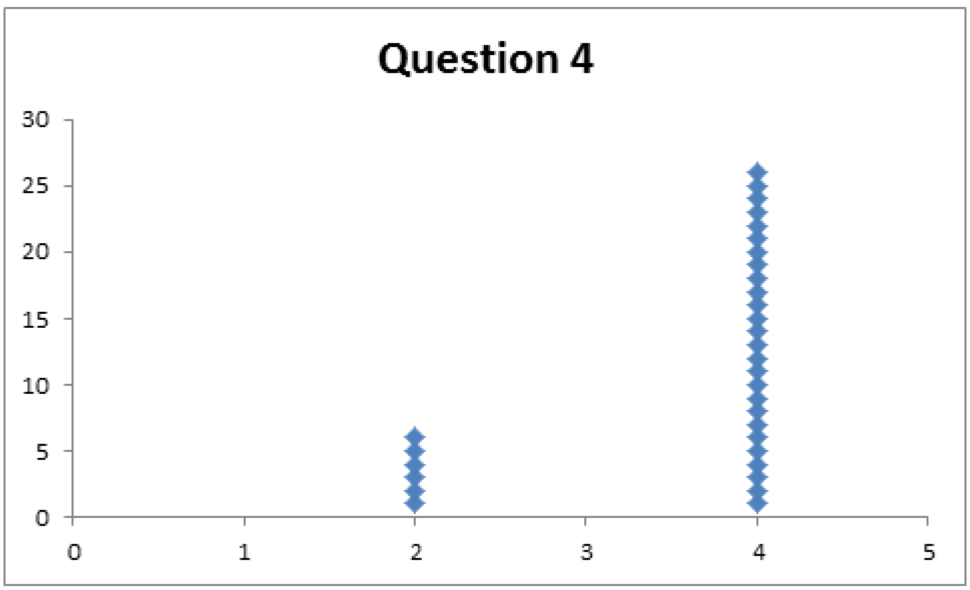
3) A complete binary tree graph has 3 levels. How many vertices does it have? (4 points)

1+2+4+8=15



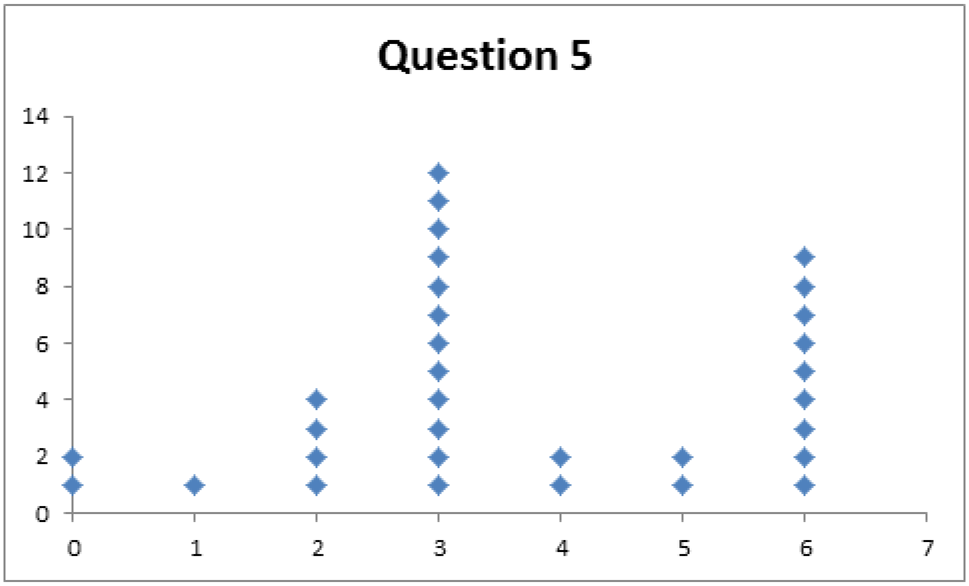
4) Sketch a graph of a complete binary tree with 3 levels. Be sure your drawing makes it obvious it's a complete binary tree. (4 points)



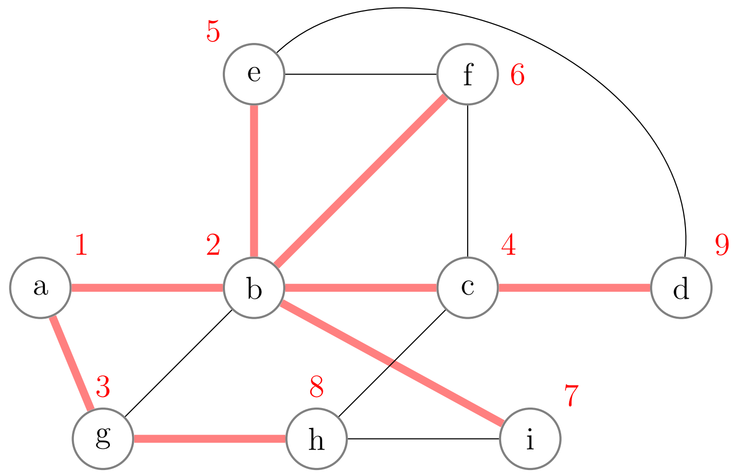


5) A "3-regular graph" is defined as graph in which each vertex has degree exactly 3. Additionally, we'll label each of the n vertices with an integer label, and store the edges in a $3 \times n$ matrix. What is the asymptotic runtime of a properly implemented depth-first search in this situation? (6 points)

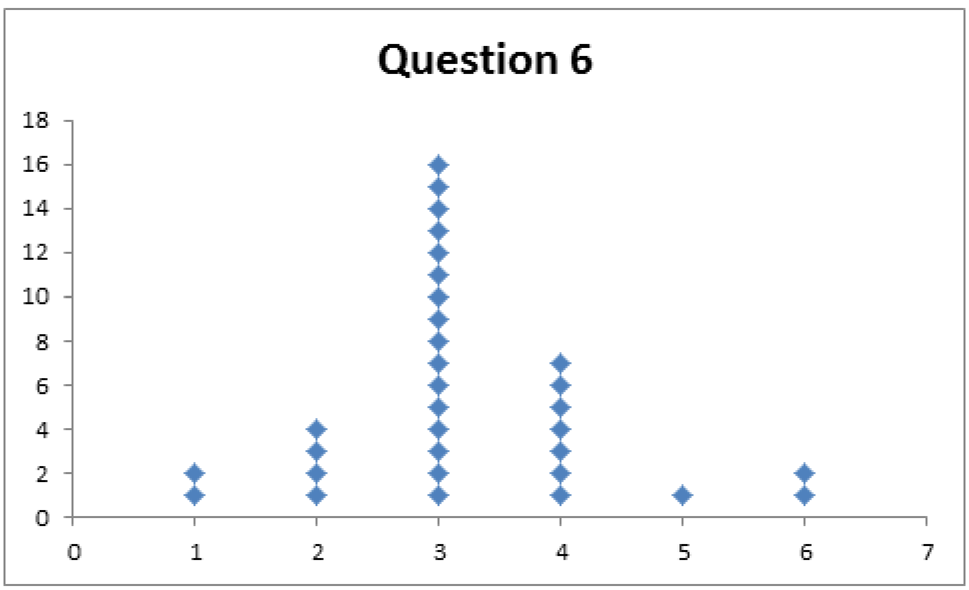
In this algorithm we iterate through the vertices, and in each case we visit at most 3 neighbors, so we get $O(3n) = O(n)$



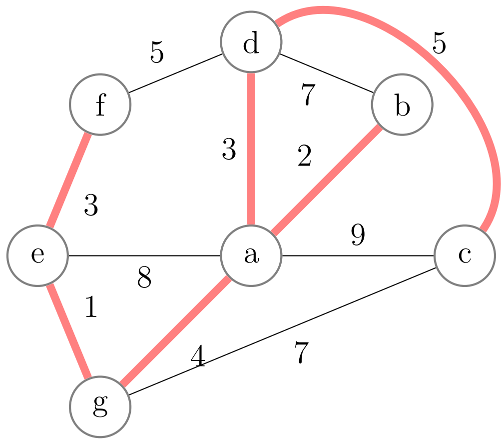
6) Find a spanning tree of the graph below, using a breadth-first construction. Label the vertices in the order in which you add them to the tree. Start from the vertex labeled ‘a’ and evaluate vertices in lexicographic order. (6 points)



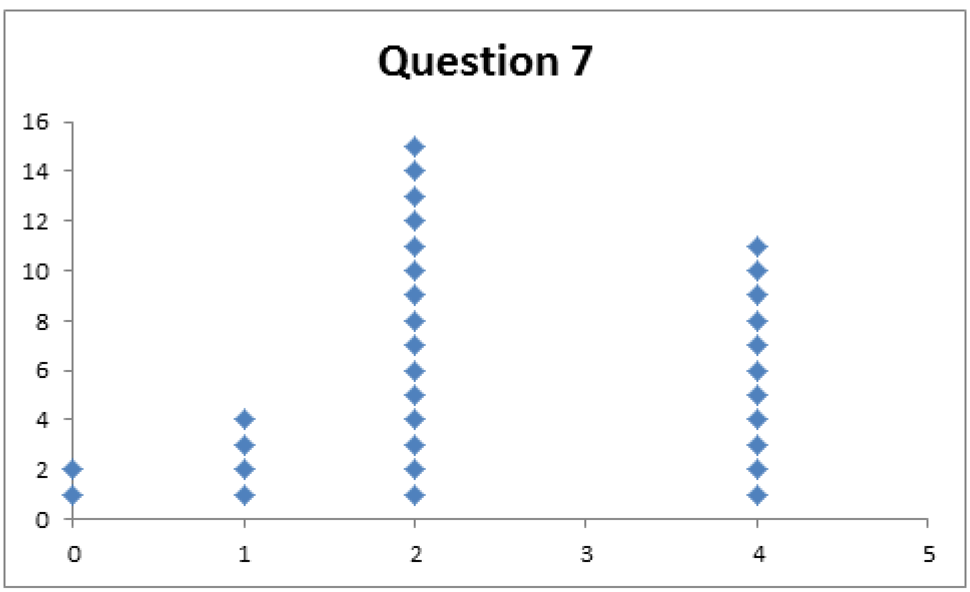
Grader’s Note: Make sure to read the instructions! Only a handful of people actually labeled the verticies. You can’t get full credit if you don’t answer the question.



7) Find a minimal spanning tree on the graph shown below. (4 points)

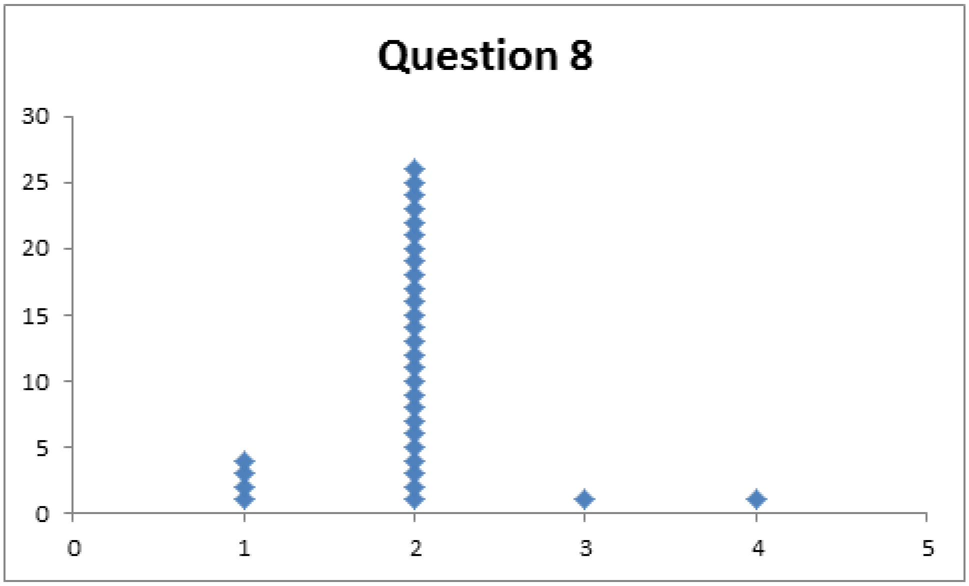


The solution is unique, but you might get to it in different ways depending on what algorithm you use.

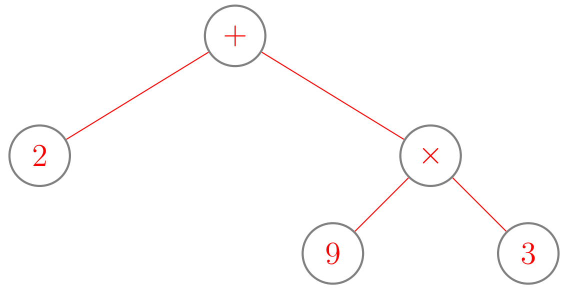


8) A tree has m leaves. Find a bound on the total number of vertices. (4 points)

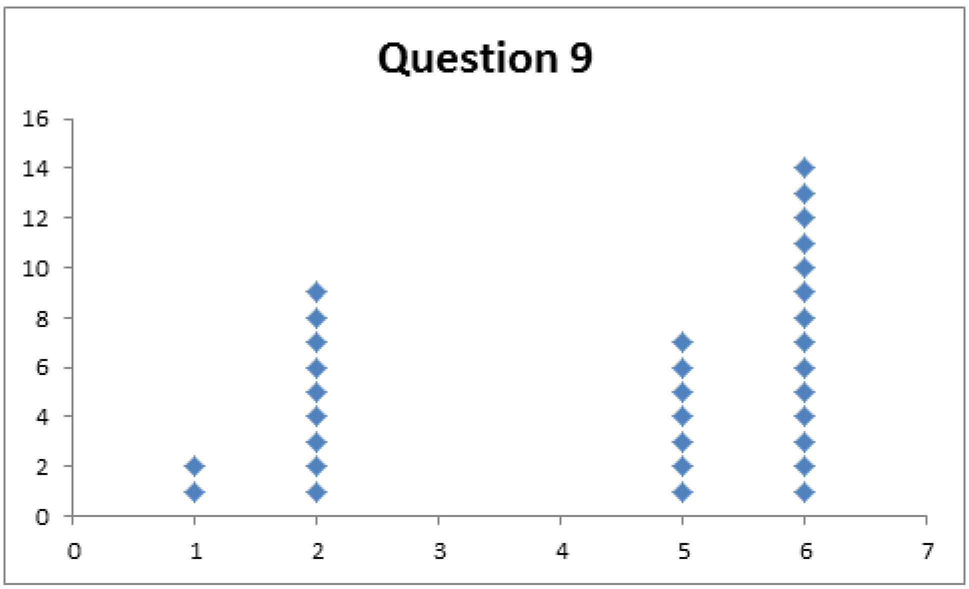
A tree could have many internal leaves, in fact a path of size n has n nodes and just 2 leaves, so there is no finite bound on the number of vertices. The best answer would be $o(\infty)$, but $O(\infty)$ was also accepted for full credit. If you put $\Omega(m)$, that's also correct but was not given full credit because it's not very meaningful.



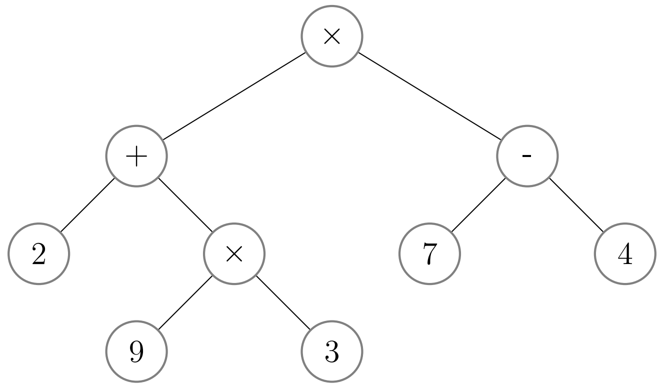
9) Construct the expression tree for the mathematical expression $2 + 3 \cdot 4$. (6 points)



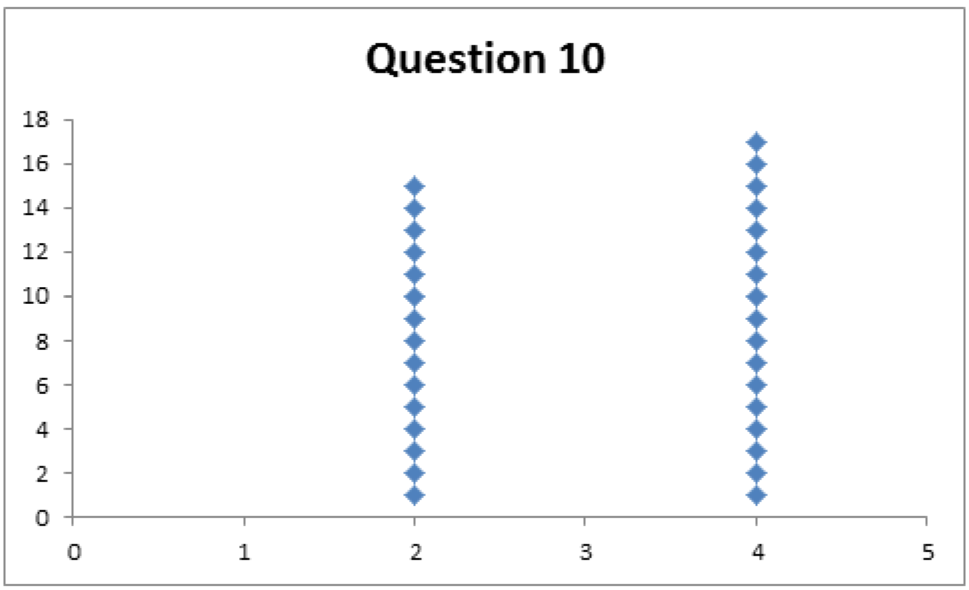
Because of the order of operations, note that the multiplication comes before the addition. If your tree does it in a different order, it's completely wrong. More partial credit was given if your tree computed the correct answer, but in a different method. For instance, many people drew the tree corresponding to $3 \cdot 4 + 2$.



10) Write the postfix expression corresponding to the expression tree below. Do not evaluate it. (4 points)

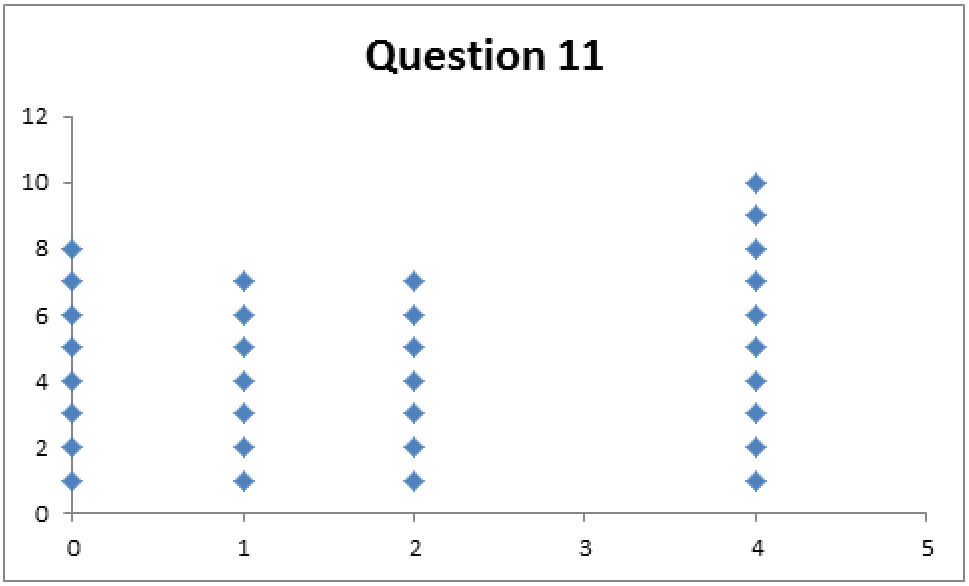


$293 \times +74 - \times$



11) In a StarCraft tournament, there are n contestants. The contestants, in pairs, play one game against each other and there is always a winner. Find and justify a bound on the total number of games required to determine the best player. Assume that skill is transitive: if a defeats b and b defeats c, then it is assumed that a could defeat c. (4 points)

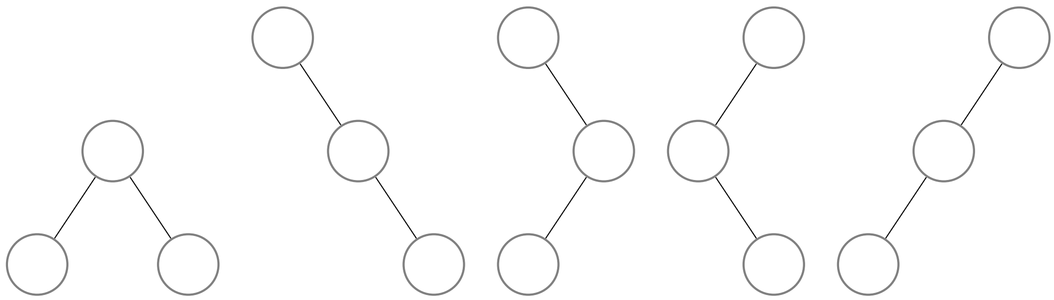
$O(n)$. More precisely, you have $O(log(n))$ rounds in which you have one game for every two contestants. If there are 2^k contestants, this works out to $\frac{2^k}{2} + \frac{2^{k-1}}{2} + \cdots + 8 + 4 + 2 + 1 = 2^k - 1$, which is one less than the input size.



— End of credit-bearing portion of the quiz. The questions below are listed for review purposes only, but bear no credit on this quiz —

12) Choose one: Sketch all binary trees with 3 vertices OR how many binary trees are there on 5 vertices? (0 points)

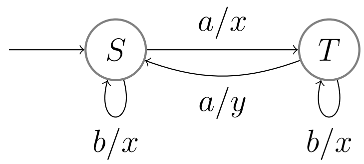
There are 5 binary trees with 3 vertices:



There are lots of binary trees with 5 vertices. Recall that this is actually the 5th Catalan number. The derivation takes some work, but it's in section 9.8 of the text. $C_5 = \frac{1}{6}\binom{10}{5} = 42$

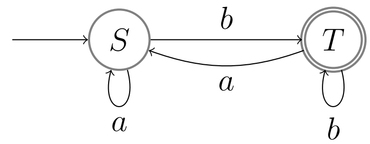
13) Sketch a transition diagram for a finite state machine with 2 states S and T, 2 input values a and b, and 2 output values x and y. (0 points)

There are many possible answers here. Below is one example of such a transition diagram.



14) Sketch a transition diagram for a finite state automata that accepts all strings over the alphabet $\{a, b\}$ that end in b. (0 points)

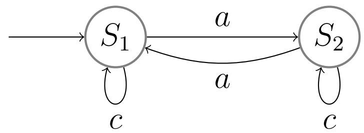
There are many possible answers here. Below is one example of such a transition diagram.



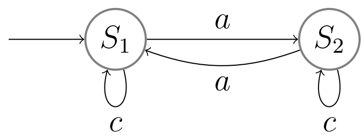
15) Give a derivation to show that $abbc$ is in the grammar $G = \{N, T, P, \sigma\}$ defined below.
(0 points)
$N = \{S, U\}$
$T = \{a, b, c\}$
$P = \{S \to a|b|c|U, U \to aUU|bUU|cUU|c, UU \to c, abUUU \to b\}$
$\sigma = S$

$S \Rightarrow U \Rightarrow aUU \Rightarrow aaUUU \Rightarrow aabUUUU \Rightarrow abU \Rightarrow abbUU \Rightarrow abbc$

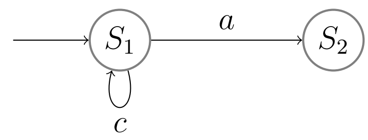
16) Given the deterministic finite state automata below, find the corresponding nondeterministic finite state automata. (0 points)



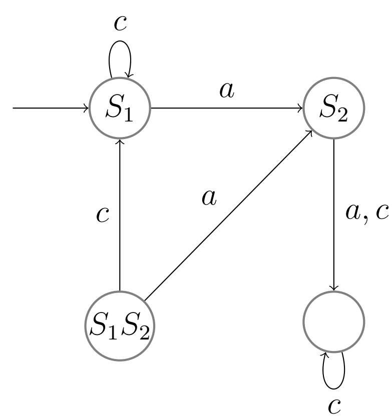
Every deterministic finite state automata is also nondeterministic finite state automata. So we may copy and paste the same thing:



17) Given the nondeterministic finite state automata below, find the corresponding deterministic finite state automata. (0 points)



This one takes a bit of work. Below we have a deterministic finite state automata that produces the same language:



Also of curious note is that for some reason I didn't give any accepting states. So this finite state automata is kind of stupid insomuch as it rejects everything. If we modified the problem to have an accepting state as below, we would get the corresponding answer.