

Name _____ Discrete II, 2/29/2016, Test 2

1) List the first seven 3-combinations from the set $\{a, b, c, d, e\}$ in lexicographic order.
(6 points)

2) List all permutations from the set $\{3, 6, 7\}$ in lexicographic order. (6 points)

3) You flip a coin 9 times. What is the probability of getting three heads and the rest tails?
(6 points)

4) A card is selected at random from a standard deck of 52 cards. What is the probability that it is a clubs? (4 points)

5) A standard deck of 52 cards is sitting on a table. You draw 2 cards. What is the probability that the second card you draw is a queen? (4 points)

6) Two standard dice are rolled. What is the probability that the two numbers sum to 8?
(6 points)

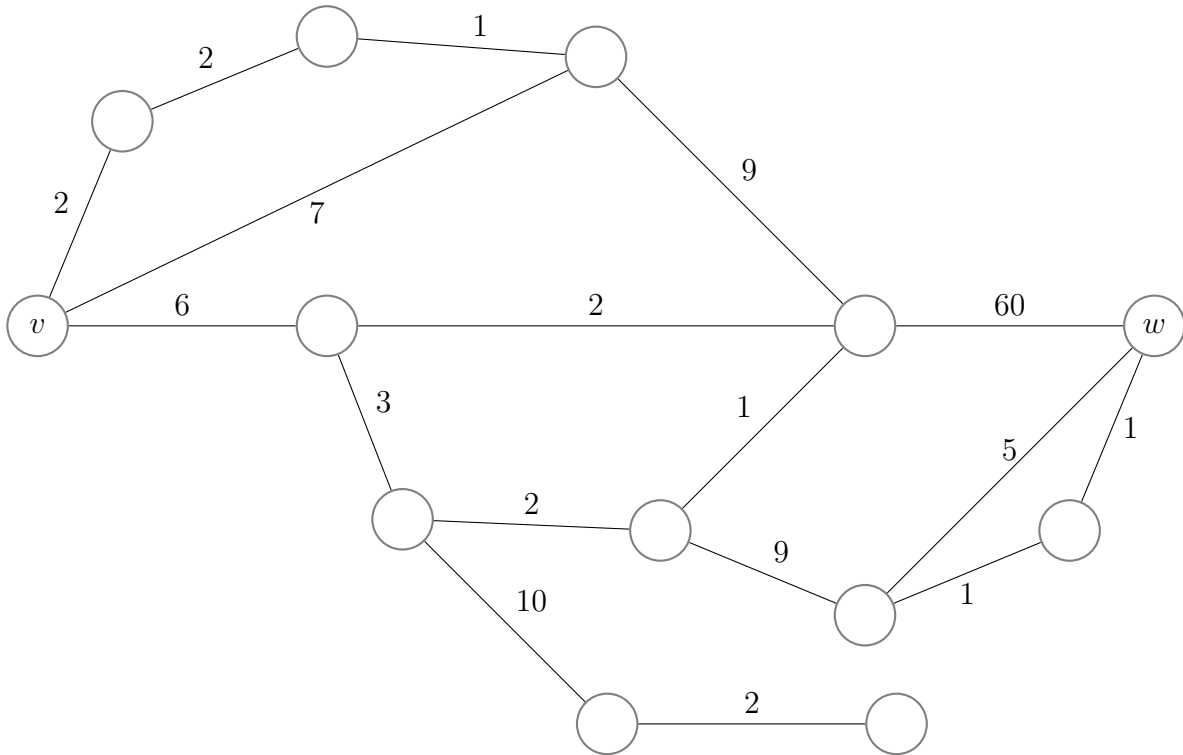
7) Write down $(a + b)^{38}$ as a summation of each of the 39 terms. (4 points)

8) What is the term containing x^{14} in $(x + 2y)^{50}$? (6 points)

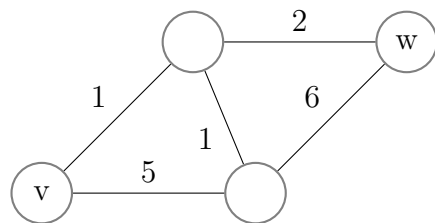
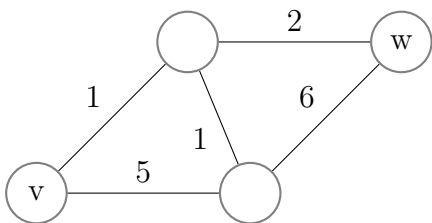
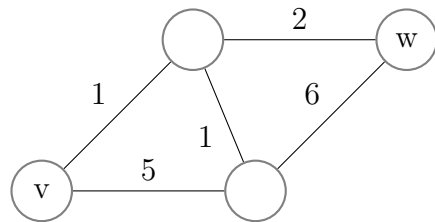
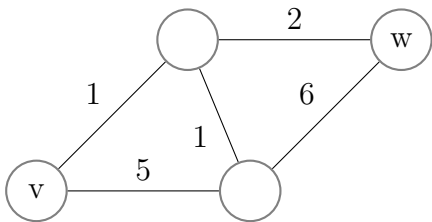
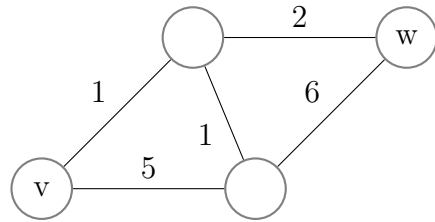
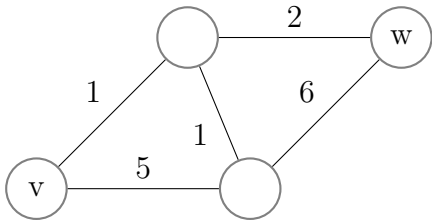
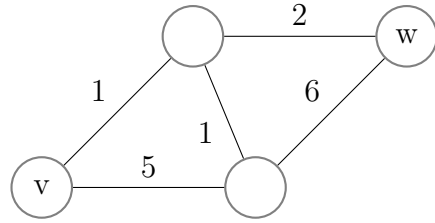
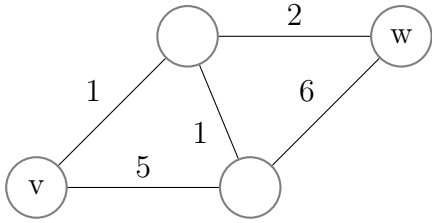
9) Draw an example of a path with 5 vertices (4 points)

10) Sketch a graph with 8 vertices in which each vertex has degree 3. (4 points)

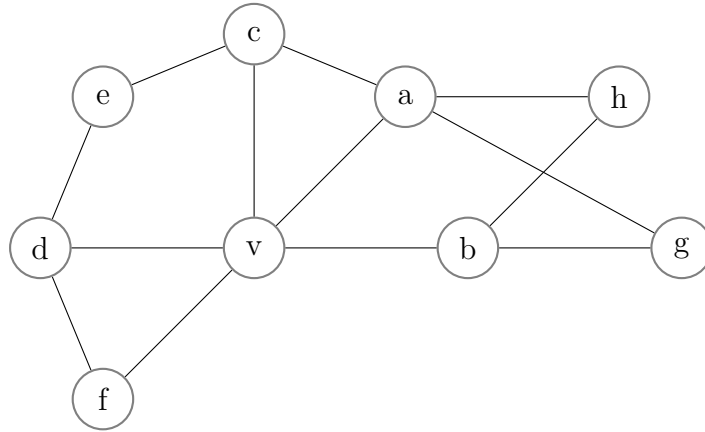
11) On the graph below, use Dijkstra's algorithm to find the shortest path from v to w . Please effectively illustrate how the algorithm works. (10 points)



12) Below is a graph (actually 8 drawings of it). You can run Dijkstra's algorithm to find the shortest path from v to w - do so, using each new graph to illustrate one new step. You might not need all 8 copies of the graph. (8 points)



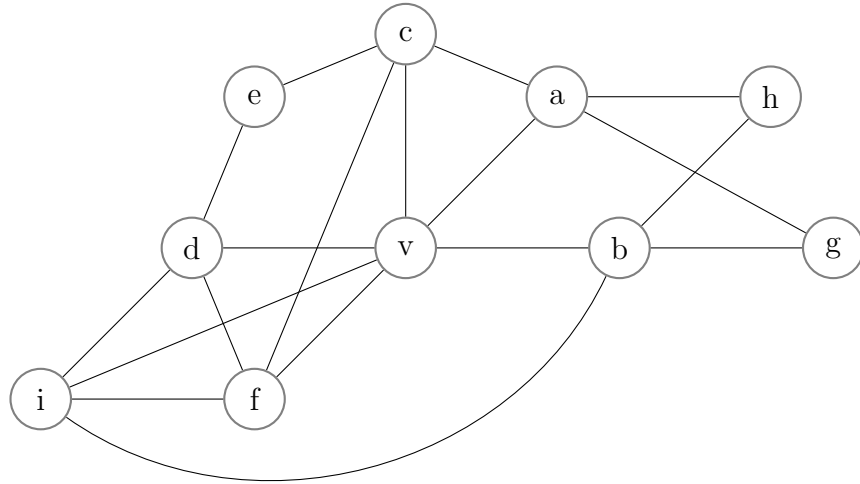
13) In the graph below, identify a bipartite subgraph. Illustrate why the subgraph you chose is bipartite. (6 points)



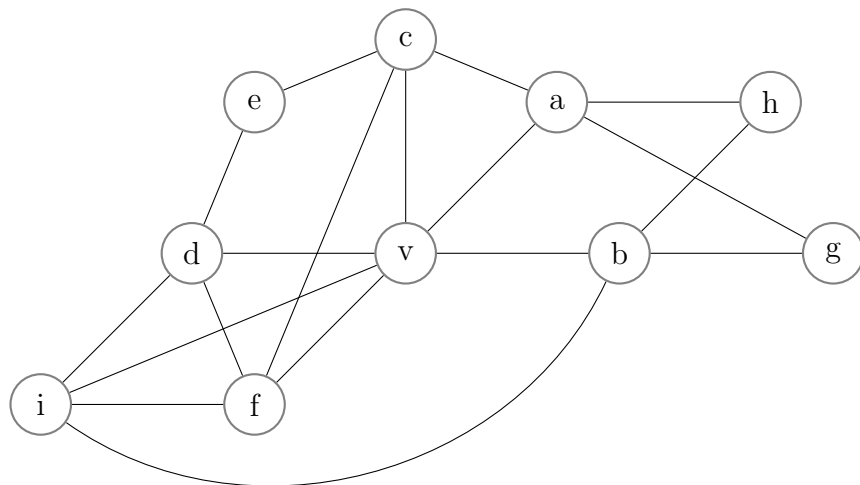
14) Sketch a graph with the incidence matrix given below. (4 points)

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

15) Using the graph below, either find and label a Hamiltonian cycle, or give a reason for why one does not exist. (6 points)



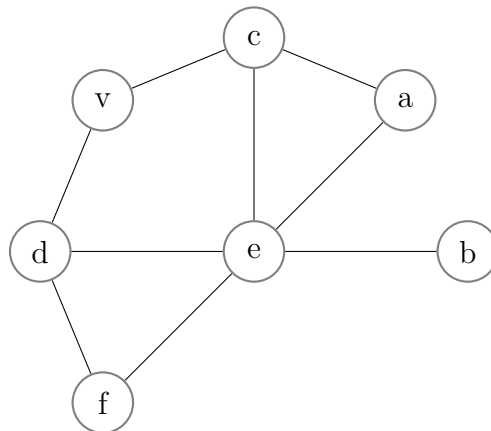
16) Using the graph below, either find and label an Eulerian tour, or give a reason for why one does not exist. (6 points)



17) Sketch two graphs, each with 6 vertices, that are not isomorphic. (3 points)

18) Sketch two graphs, each with 6 vertices, that are isomorphic. (3 points)

19) In the graph below, what is the degree of v ? (4 points)



20) **Bonus Question.** Let G be a graph with n vertices and m edges. Using proper notation, give an obvious lower bound on the runtime needed to find an Eulerian tour. (2 points)