Name ____ Discrete II, 4/13/2016, Test 3

1) A planar graph has 5 faces and 11 edges. How many vertices does it have? (4 points)

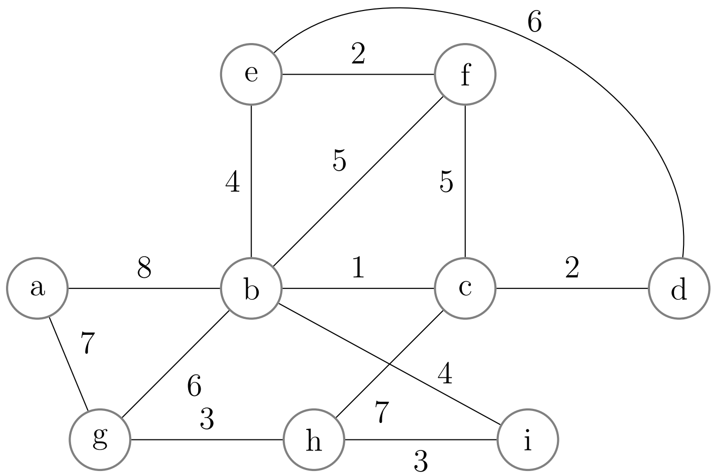
2) Draw a graph that is not planar. (4 points)

3) Sketch a graph of a complete binary tree with 7 vertices. Be sure your drawing makes it obvious it's a complete binary tree. (4 points)

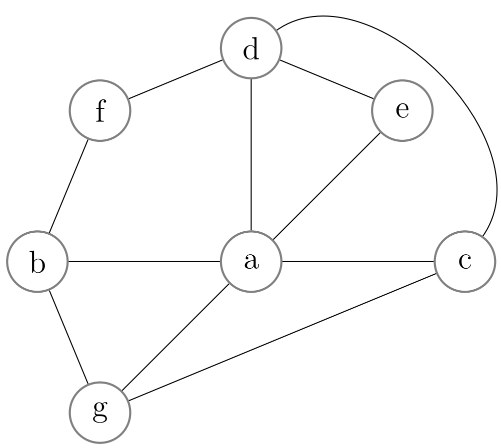
4) What is the asymptotic number of edges for a planar graph with n vertices? (4 points)

5) Given a graph G with n vertices and m edges, how many vertices and edges does a spanning tree of G have? Note that the answer to this question has two parts: (1) number of vertices and (2) number of edges. (4 points)

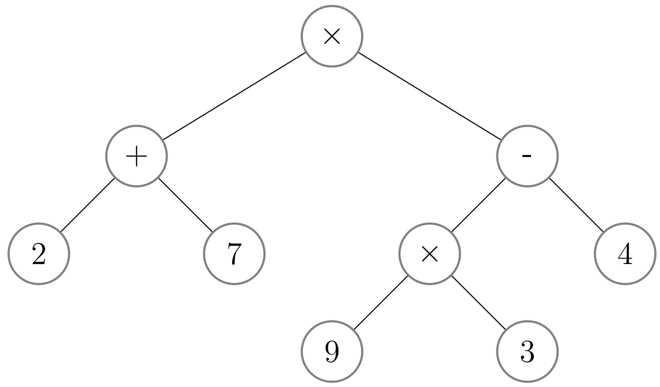
6) Find a minimal spanning tree on the graph shown below. Label the edges in the order in which you add them to the tree. (6 points)



7) Find a spanning tree of the graph below, using a depth-first construction. Label the vertices in the order in which you add them to the tree. Start from the vertex labeled 'a' and evaluate vertices in lexicographic order. (6 points)



8) Write the infix expression corresponding to the expression tree below. Do not evaluate it. (4 points)



9) In this problem a fictitious game called ”n-ary tic-tac-toe” is explained, and then you will be asked to find a bound on the size of the decision tree. Players will alternate turns. Every turn the player may make one of 9 moves. To clarify, every single turn a player has 9 options. After n moves, the game finishes and a winner is declared. Find an asymptotic upper bound for the size of the decision tree that would represent this game. (4 points)

10) Construct the expression tree for the prefix mathematical expression $- + 4 \times 325$. All numbers listed here are single-digit numbers. (6 points)

11) In the game "Love Letter" two players alternate playing cards from their hand until all cards are played. Each player starts with 6 cards in his or her hand. We want to see what the decision tree looks like, but to simplify the problem for this test, let us assume that each player starts with only 3 cards in his or her hand instead of 6. Sketch a graph of the first three levels of the decision tree. Do not count the root as a level. You do not need to label the vertices. (4 points)

12) Sketch four non-isomorphic binary trees with 4 vertices (4 points)

13) Sketch a transition diagram for a finite state machine with 3 states S, T, and U, 2 input values a and b, and 2 output values x and y. (4 points)

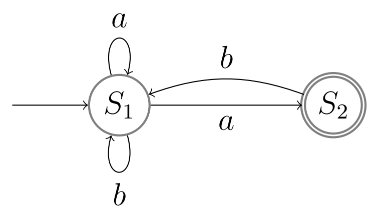
14) Use the finite state machine you drew in the previous problem for this problem. Give the output corresponding to the input $abaaba$ (6 points)

15) Sketch a transition diagram for a finite state automata that accepts only the string *abcd* and nothing else. (4 points)

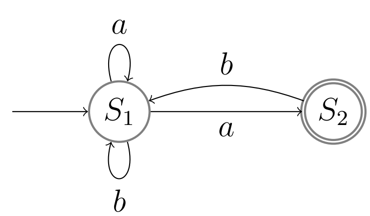
16) Give a derivation to show that *abab* is in the language defined by the grammar $G = \{N, T, P, \sigma\}$ below. (6 points)
$N = \{\sigma, A, B\}$
$T = \{a, b, c\}$
$P = \{\sigma \to AB, A \to aA|a, B \to Bb|b, AB \to BA\}$
$\sigma = \sigma$

17) True or false: The language created by the grammar in the previous problem is context-free. (4 points)

18) Given the nondeterministic finite state automata below, find the corresponding deterministic finite state automata. (6 points)



19) Given the nondeterministic finite state automata below, illustrate why the string "*baa*" is accepted. (4 points)



20) True or false: Given the nondeterministic finite state automata in the previous problem, the string "*baba*" is accepted. (4 points)

21) A tree has y internal vertices. Find a lower bound on the total number of vertices. (4 points)

22) A tree has y internal vertices. Find an upper bound on the total number of vertices. (4 points)