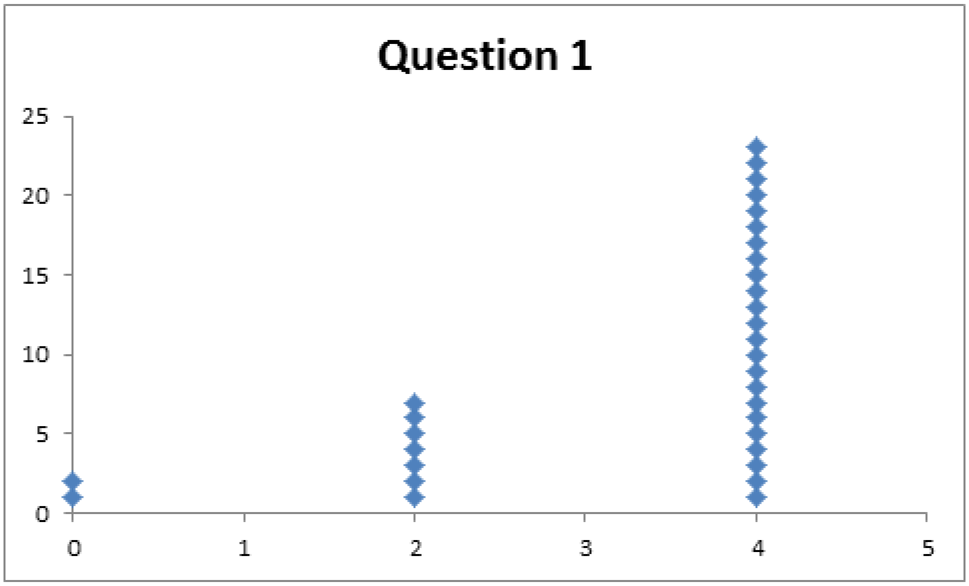
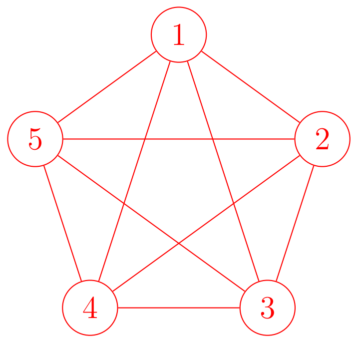
Name ______________________________ Discrete II, 4/13/2016, Test 3

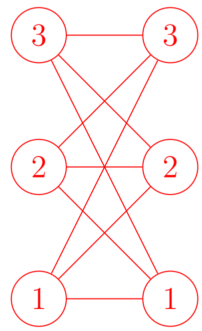
1) A planar graph has 5 faces and 11 edges. How many vertices does it have? (4 points)
Recall the formula $f = e - v + 2$, plugging in our values we get $5 = 11 - v + 2$. Then solve for v to get: $v = 8$. If all else fails, graph a particular graph and count the vertices. No matter how you graph it, it will always have 8 vertices.

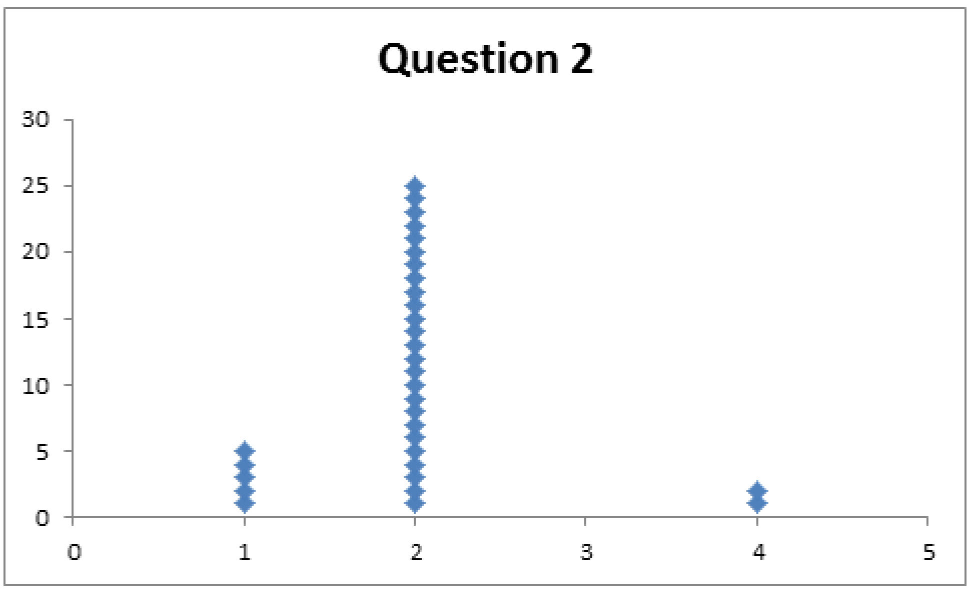


2) Draw a graph that is not planar. (4 points)

There are many possible answers here. Just make sure the graph is planar, not just your drawing. The two simplest non-planar graphs are K_5 and $K_{3,3}$, both of which are illustrated below.

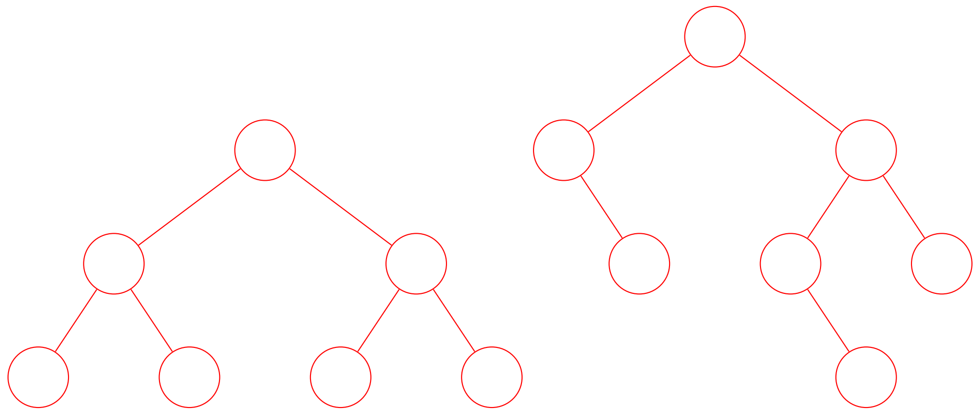


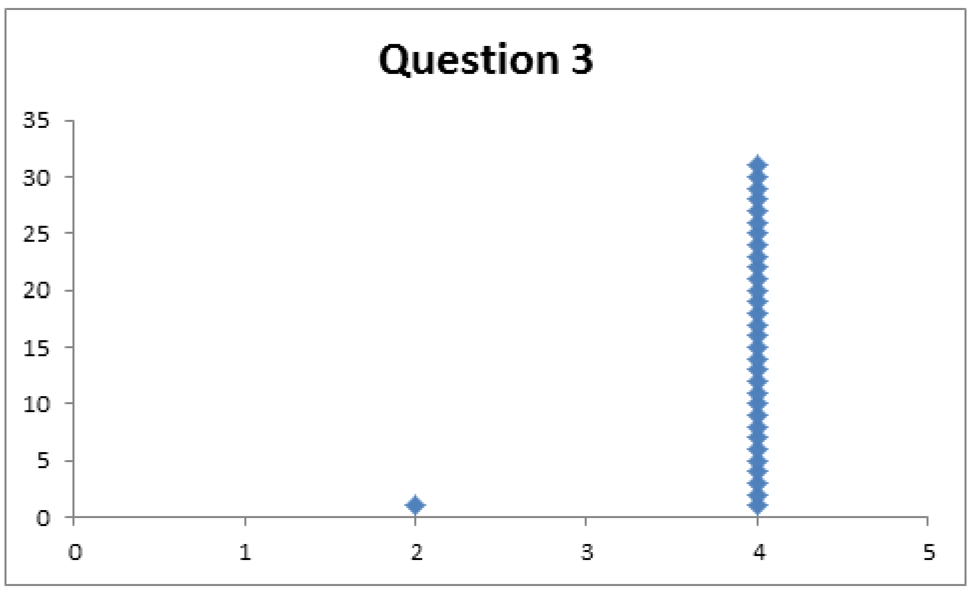




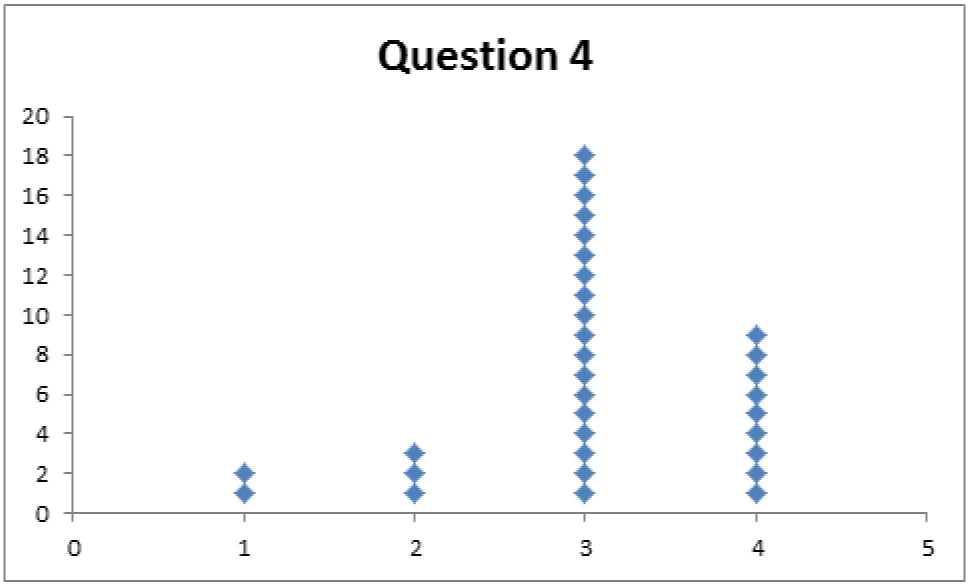
3) Sketch a graph of a complete binary tree with 7 vertices. Be sure your drawing makes it obvious it's a complete binary tree. (4 points)

There is only one complete binary tree with 7 vertices, shown below to the left. Everything else, such as the one on the right is incomplete.



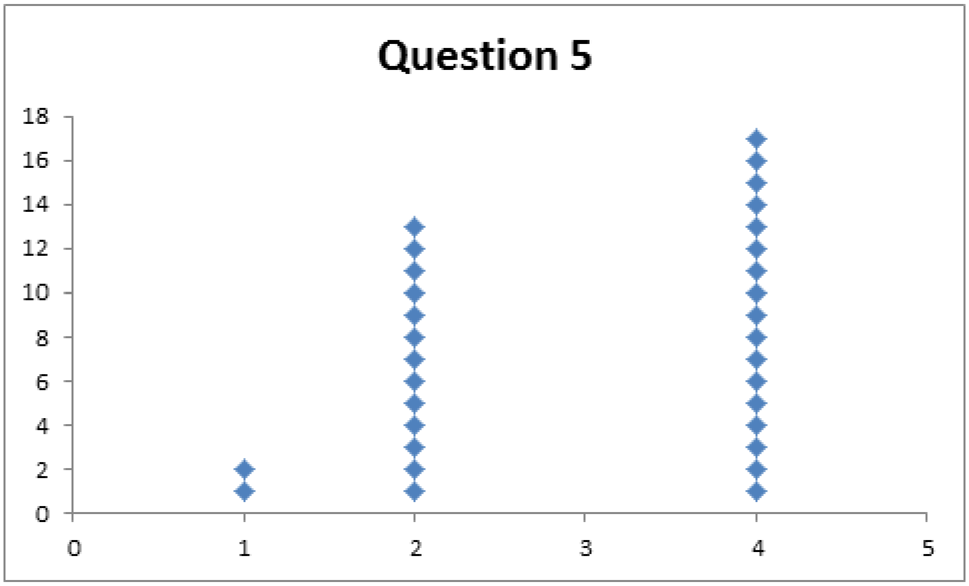


4) What is the asymptotic number of edges for a planar graph with n vertices? (4 points)
$O(n)$ or $O(n + f)$ where f is the number of faces. Actually these are equal because $O(f) = O(n)$ as well.



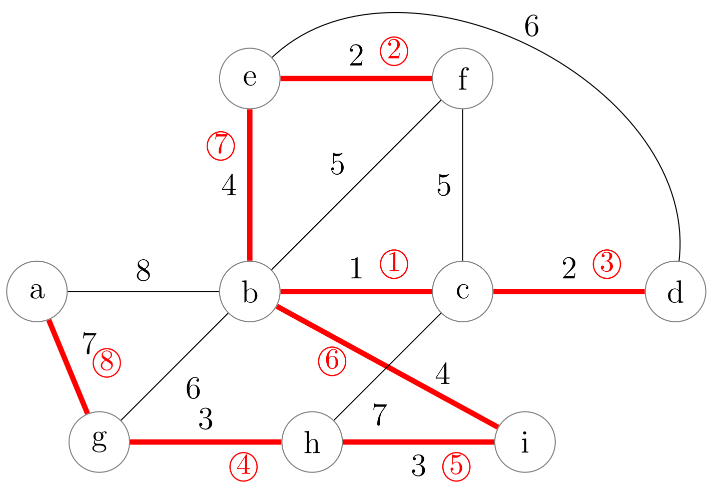
5) Given a graph G with n vertices and m edges, how many vertices and edges does a spanning tree of G have? Note that the answer to this question has two parts: (1) number of vertices and (2) number of edges. (4 points)

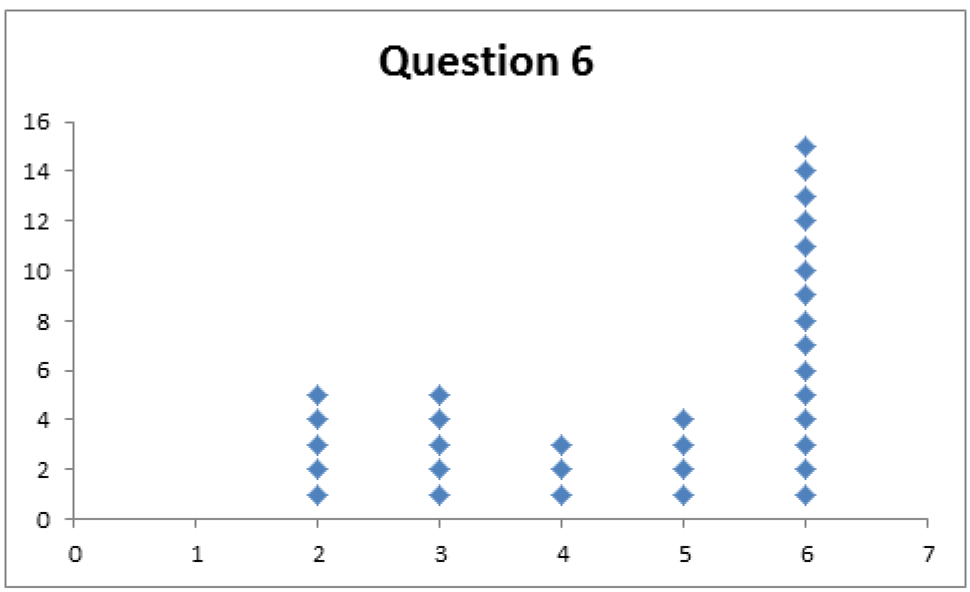
n vertices and $n-1$ edges.



6) Find a minimal spanning tree on the graph shown below. Label the edges in the order in which you add them to the tree. (6 points)

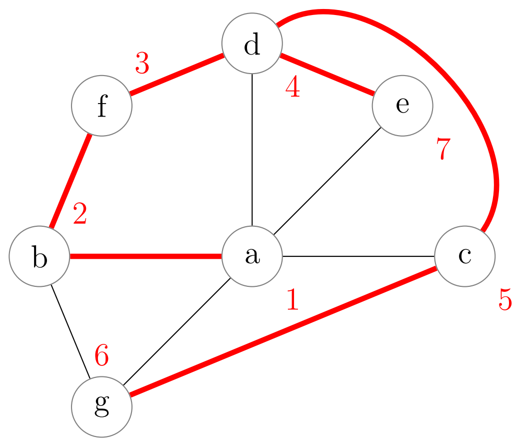
There are multiple correct answers depending on whether you used Prim's or Kruskal's algorithm and how you broke ties. Below is a solution using Kruskal's algorithm.

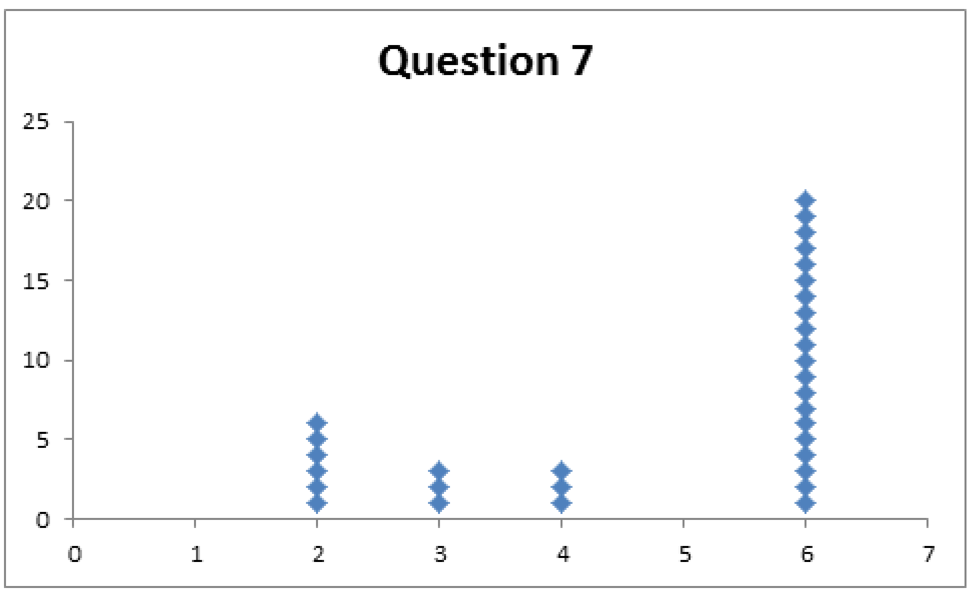




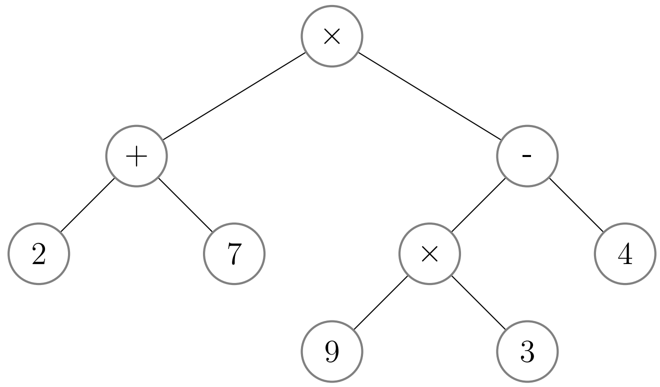
7) Find a spanning tree of the graph below, using a depth-first construction. Label the vertices in the order in which you add them to the tree. Start from the vertex labeled 'a' and evaluate vertices in lexicographic order. (6 points)

The answer here is unique. Make sure you used a depth first search.

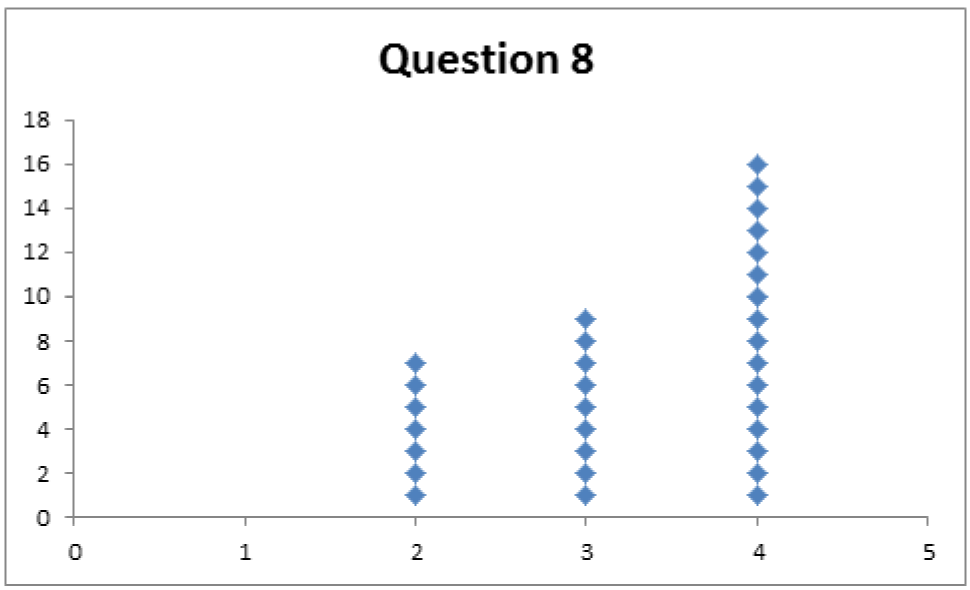




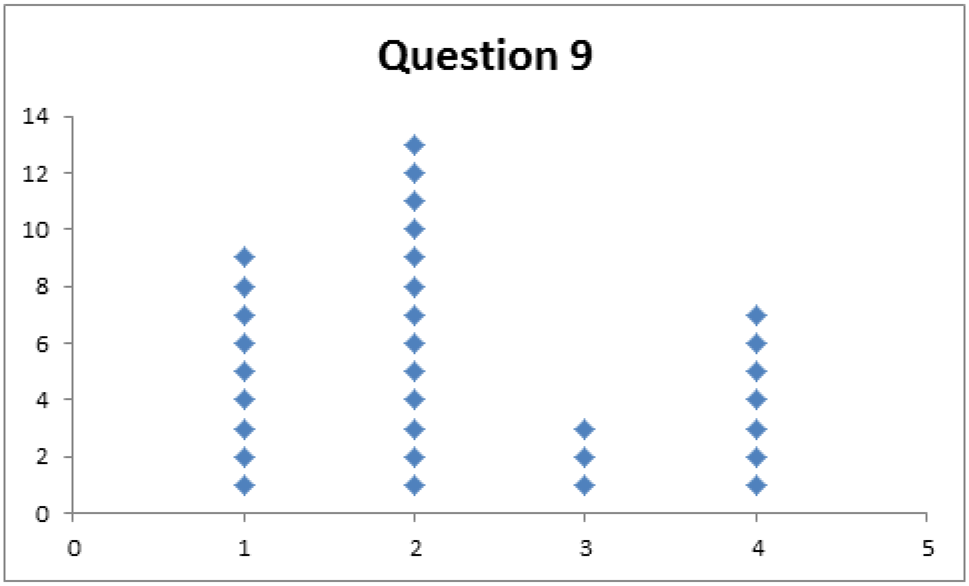
8) Write the infix expression corresponding to the expression tree below. Do not evaluate it.
(4 points)



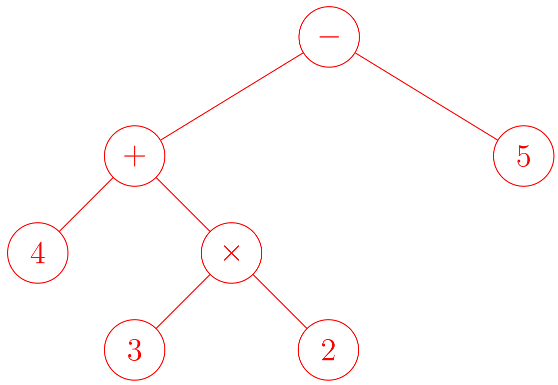
$(2 + 7) \times (9 \times 3 - 4)$

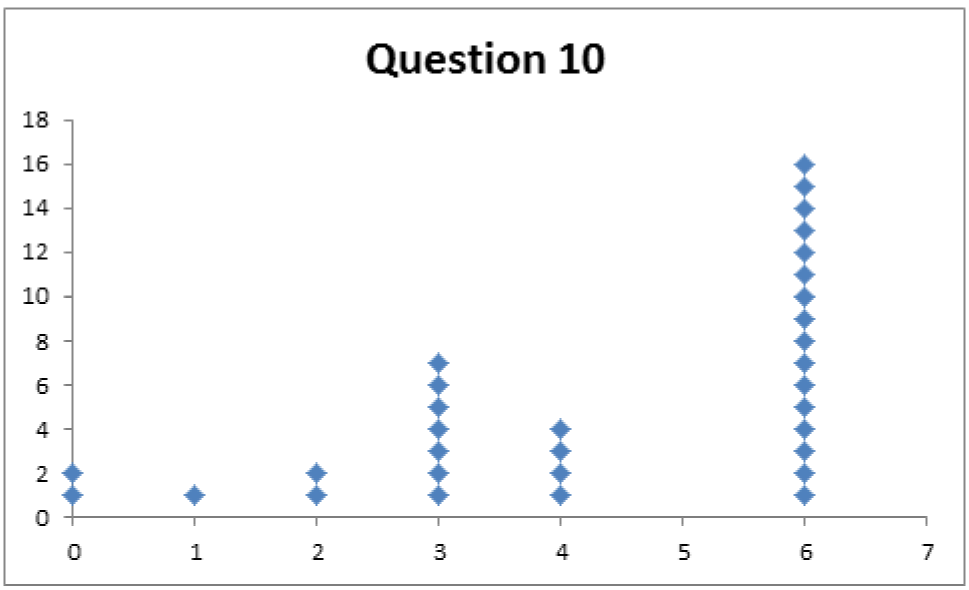


9) In this problem a fictitious game called "n-ary tic-tac-toe" is explained, and then you will be asked to find a bound on the size of the decision tree. Players will alternate turns. Every turn the player may make one of 9 moves. To clarify, every single turn a player has 9 options. After n moves, the game finishes and a winner is declared. Find an asymptotic upper bound for the size of the decision tree that would represent this game. (4 points)
$O(9^n)$

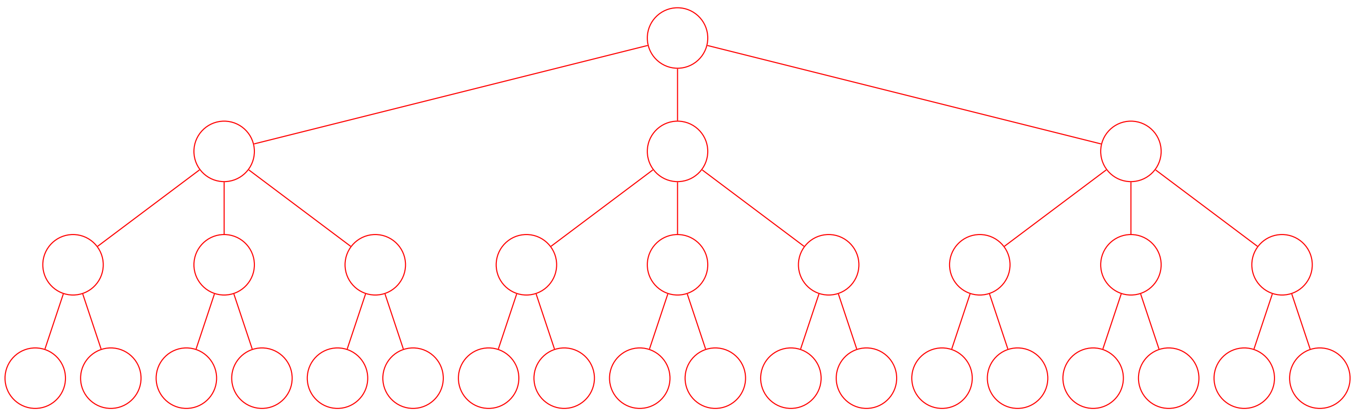


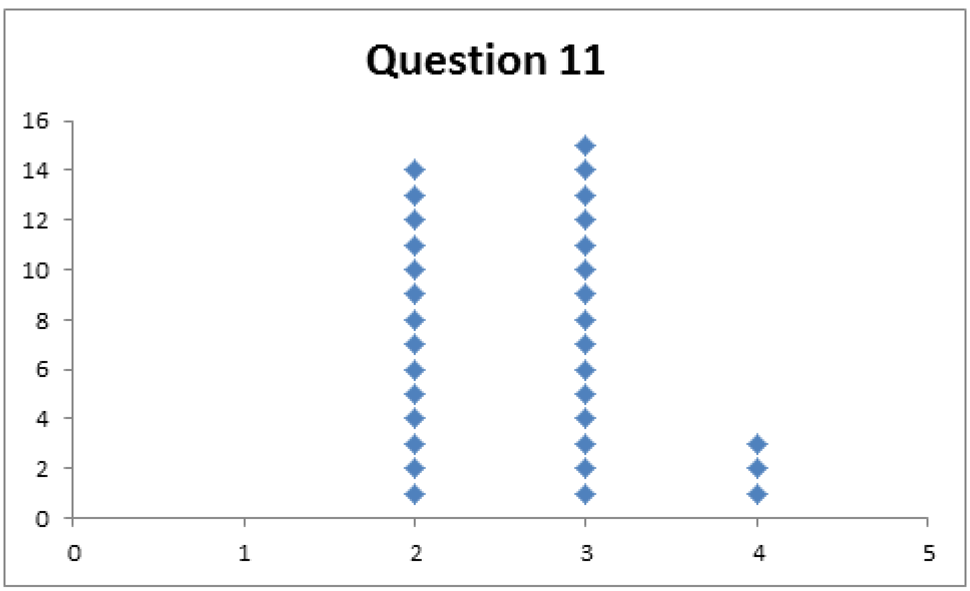
10) Construct the expression tree for the prefix mathematical expression $- + 4 \times 3 2 5$. All numbers listed here are single-digit numbers. (6 points)





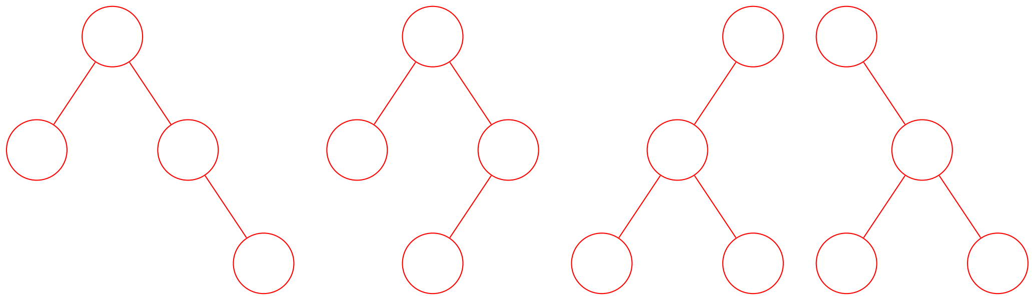
11) In the game "Love Letter" two players alternate playing cards from their hand until all cards are played. Each player starts with 6 cards in his or her hand. We want to see what the decision tree looks like, but to simplify the problem for this test, let us assume that each player starts with only 3 cards in his or her hand instead of 6. Sketch a graph of the first three levels of the decision tree. Do not count the root as a level. You do not need to label the vertices. (4 points)

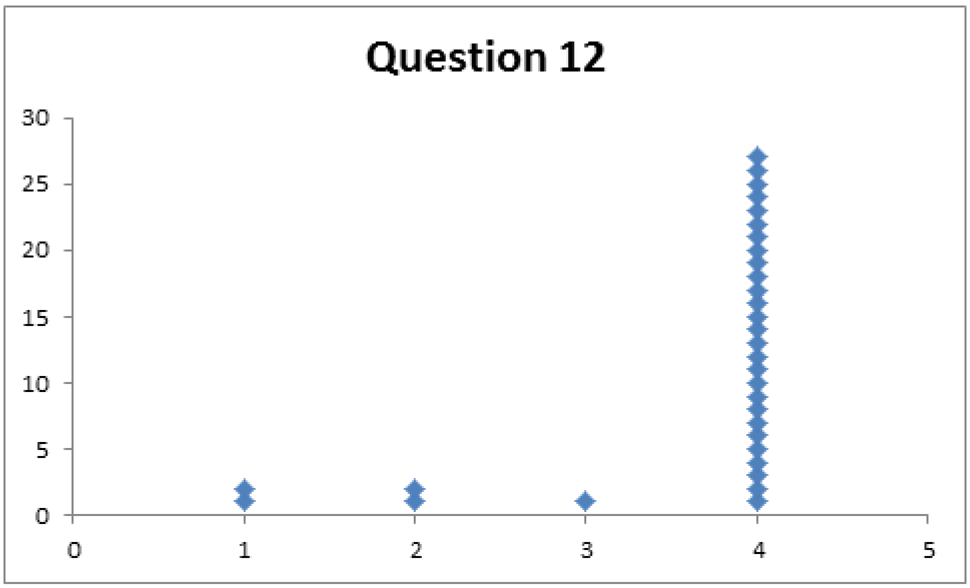




12) Sketch four non-isomorphic binary trees with 4 vertices (4 points)

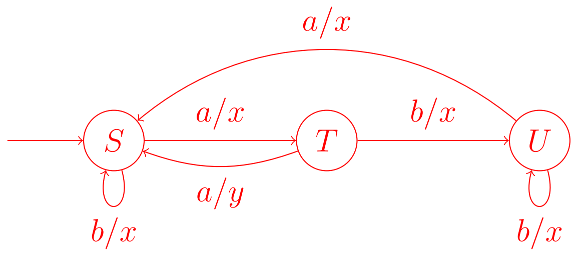
There are many solutions. Here are four examples:

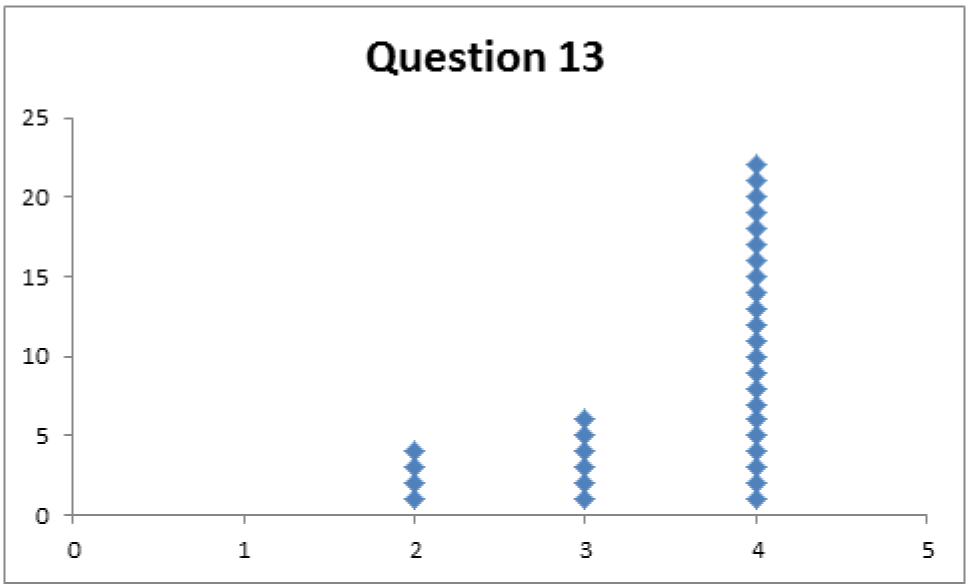




13) Sketch a transition diagram for a finite state machine with 3 states S, T, and U, 2 input values a and b, and 2 output values x and y. (4 points)

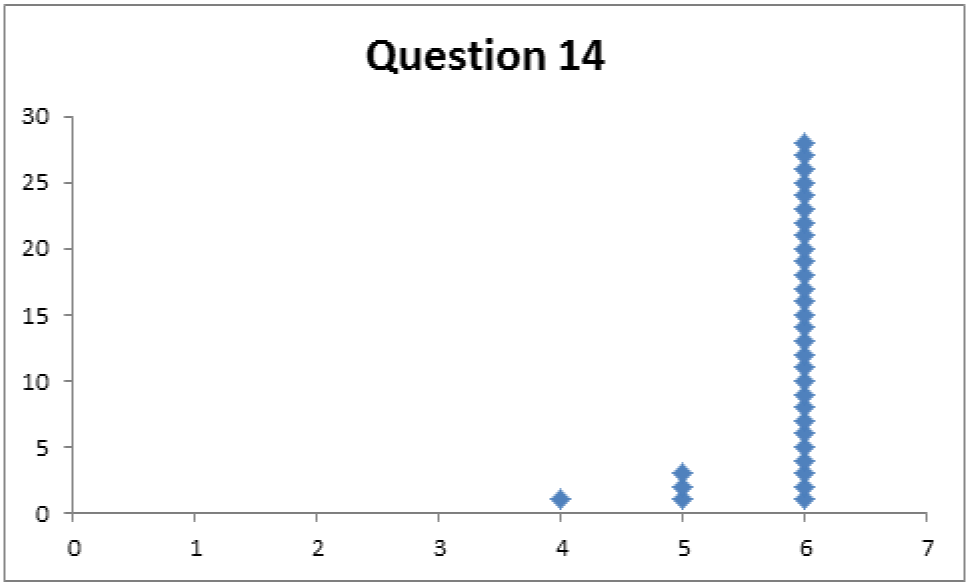
There are many possible answers here. Below is one example of such a transition diagram.





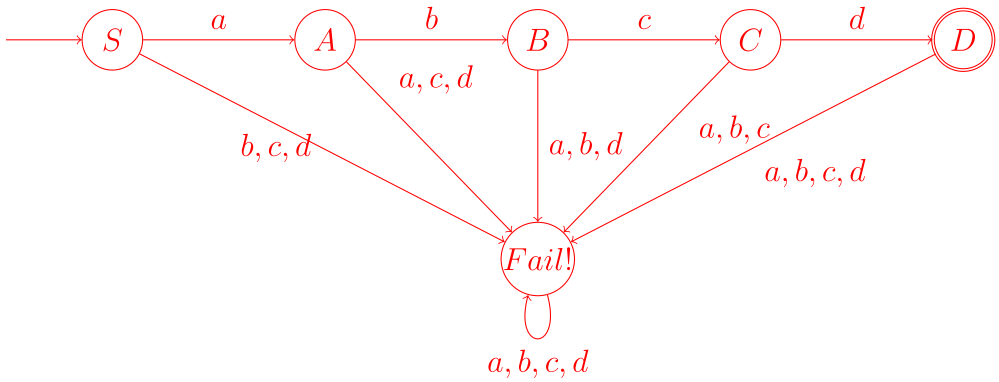
14) Use the finite state machine you drew in the previous problem for this problem. Give the output corresponding to the input *abaaba* (6 points)

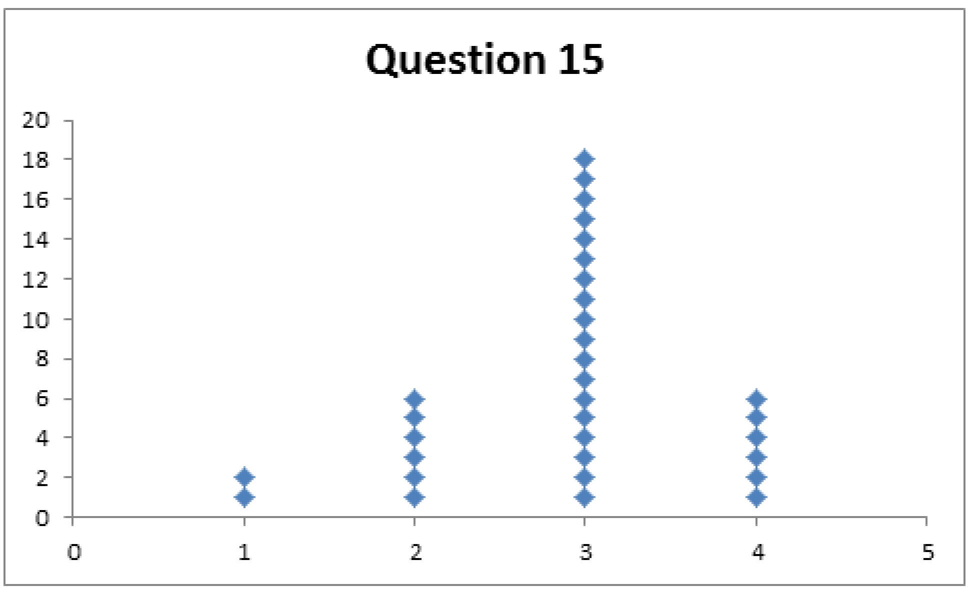
Using our FSM above, we get *xxxyxx*



15) Sketch a transition diagram for a finite state automata that accepts only the string *abcd* and nothing else. (4 points)

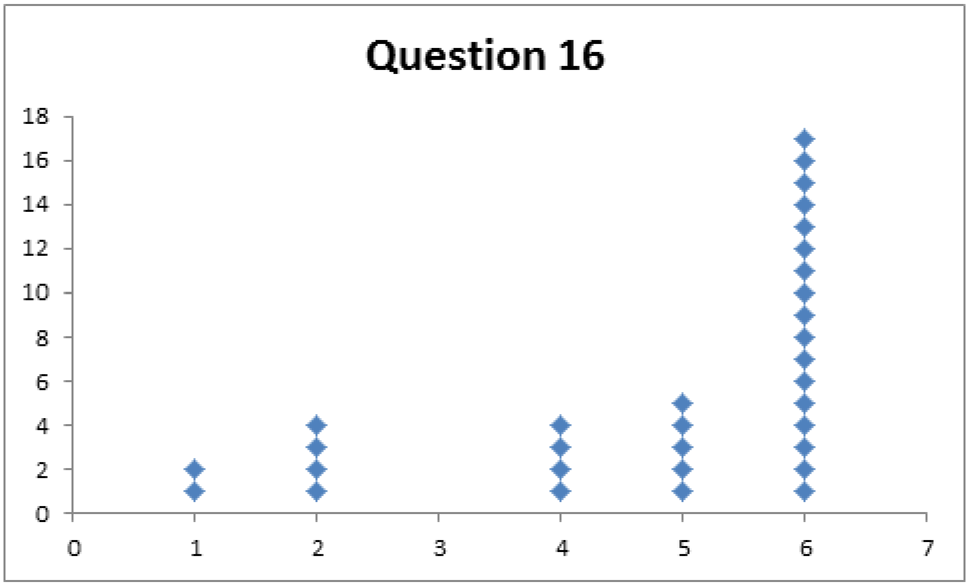
There are many possible answers here. Below is one example of such a transition diagram.



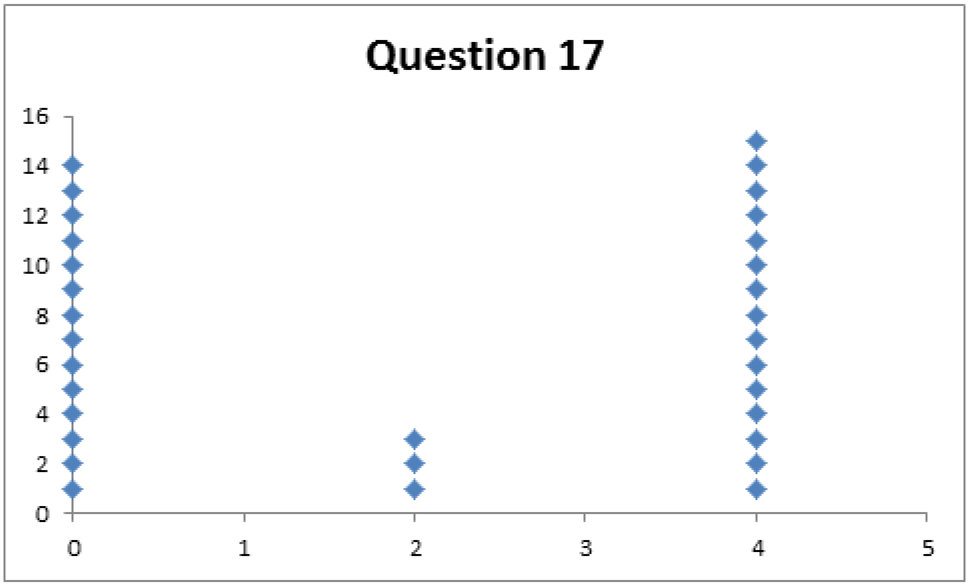


16) Give a derivation to show that $abab$ is in the language defined by the grammar $G = \{N, T, P, \sigma\}$ below. (6 points)
$N = \{\sigma, A, B\}$
$T = \{a, b, c\}$
$P = \{\sigma \to AB, A \to aA|a, B \to Bb|b, AB \to BA\}$
$\sigma = \sigma$

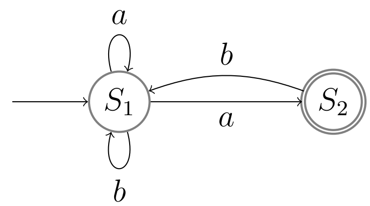
$$\sigma \Rightarrow AB \Rightarrow aAB \Rightarrow aABb \Rightarrow aBAb \Rightarrow abAb \Rightarrow abab$$

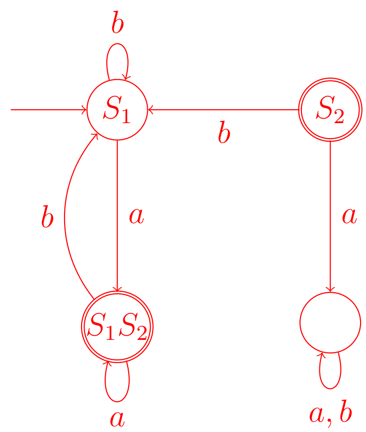


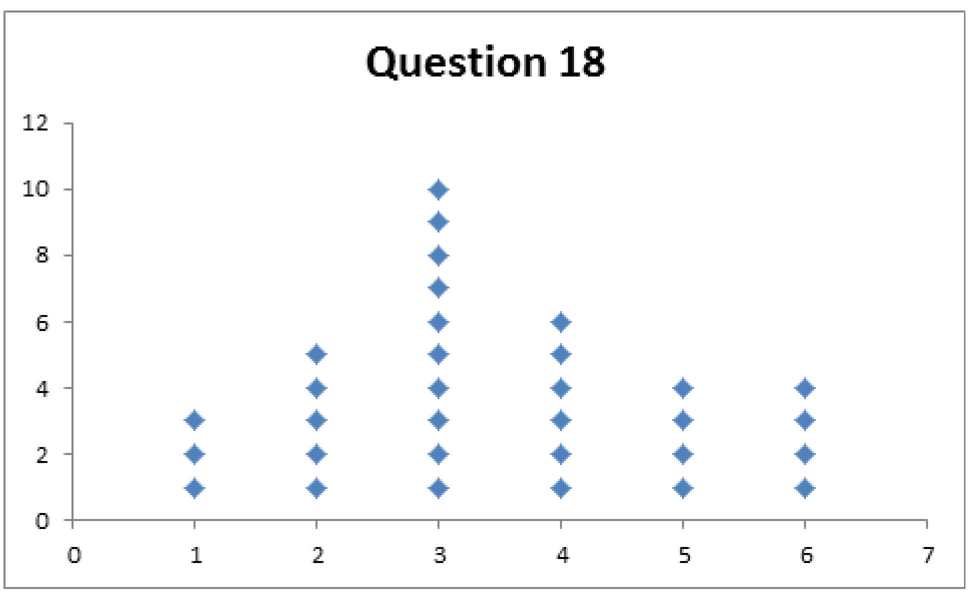
17) True or false: The language created by the grammar in the previous problem is context-free. (4 points) False



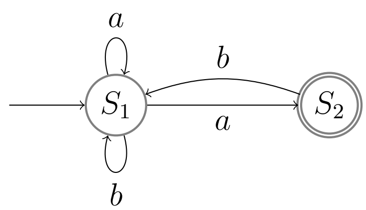
18) Given the nondeterministic finite state automata below, find the corresponding deterministic finite state automata. (6 points)

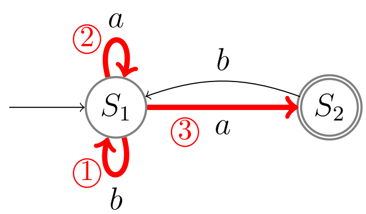


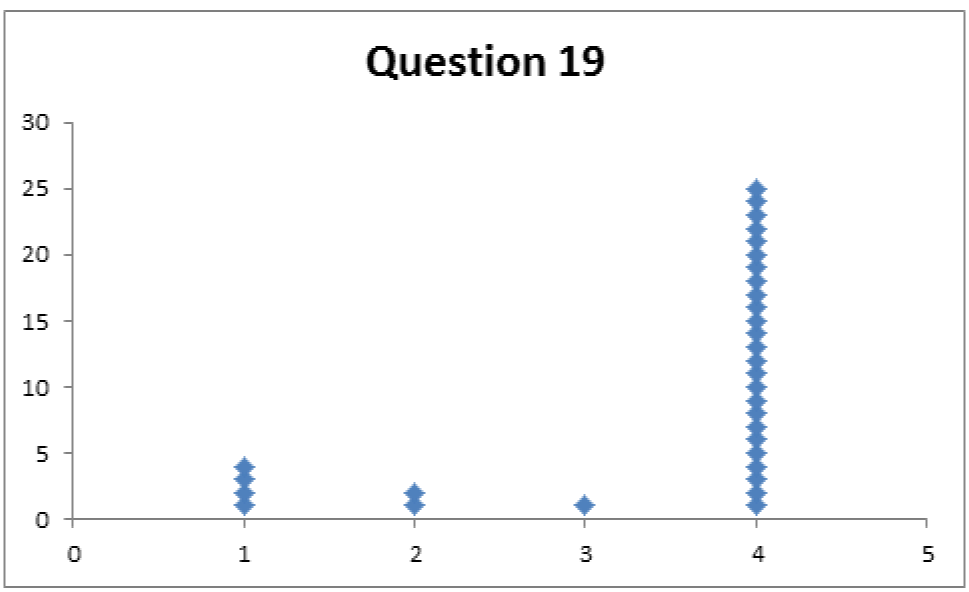




19) Given the nondeterministic finite state automata below, illustrate why the string “*baa*” is accepted. (4 points)

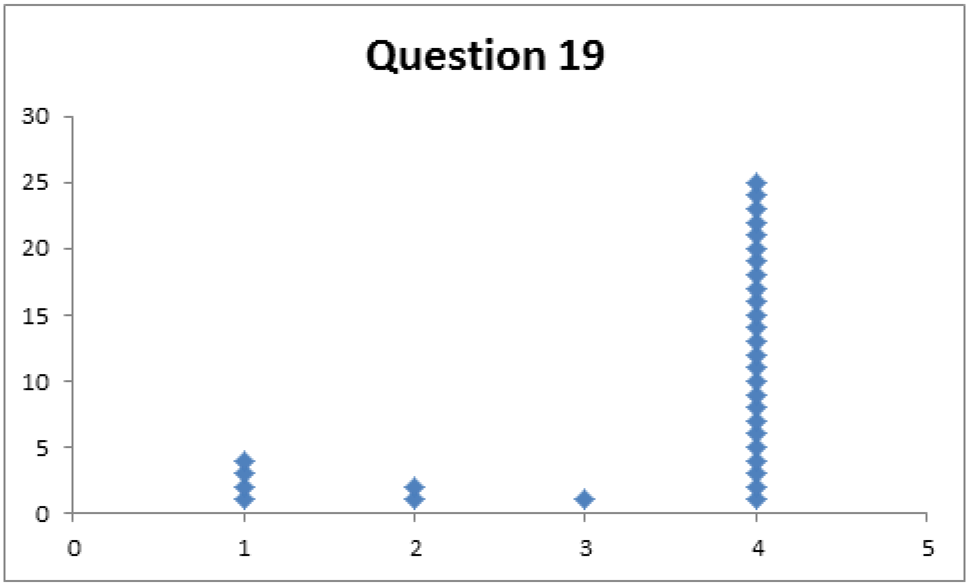






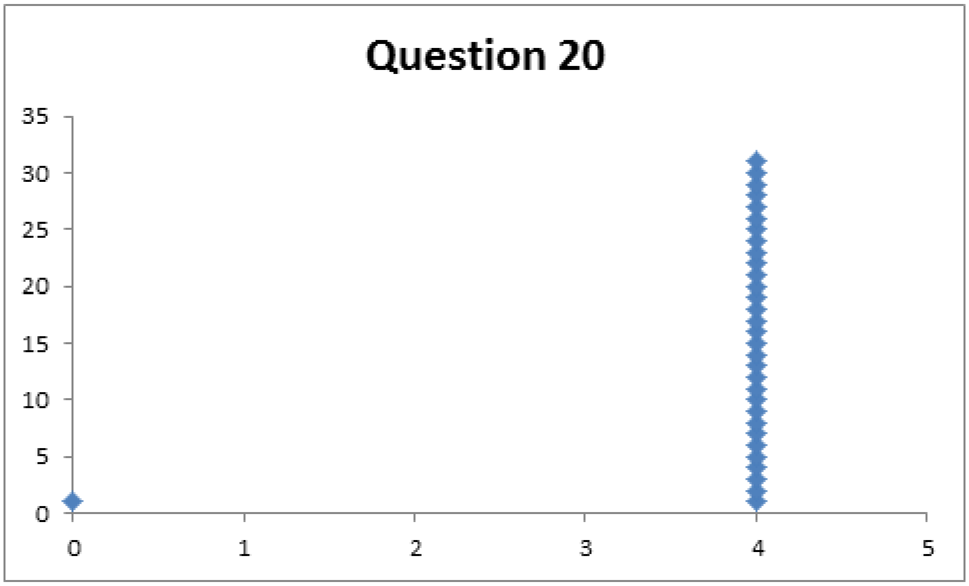
True or false: Given the nondeterministic finite state automata in the previous problem, the string "*baba*" is accepted. (4 points)

True



20) A tree has y internal vertices. Find a lower bound on the total number of vertices. (4 points)

It has at least $a + 2$ vertices (in the case of a path)



21) A tree has y internal vertices. Find an upper bound on the total number of vertices. (4 points)

The upper bound is infinity. Actually it has to be finite, but it could be arbitrarily large. So $o(\infty)$ is the best answer.