Name ______________________________________________ Discrete II, Big Quiz 1, Spring 2017

**Please show all your work and circle your answer when appropriate. You do not need to simplify answers unless the problem specifies to do so.**

1) Given $f(n) = n^3 + 3n^2 + n\log(n)$, find a big-Oh notation that gives an asymptotic upper bound for $f(n)$. (3 points)

$$O(n^3)$$

2) Given $f(n) = n^3 + 3n^2 + n\log(n)$, find a big-Omega ($\Omega$) notation that gives an asymptotic lower bound for $f(n)$. (3 points)

$$\Omega(n^3)$$

3) Justify the claim that $3n^2 + n$ is $O(n^2)$. (4 points)

$$3n^2 + n \le 3n^2 + n^2 = 4n^2 \text{ is } O(n^2)$$

4) How many 12-bit strings can be formed from the letters $A, B, C$, and $D$? (3 points)

$$4^{12}$$

5) Suppose a bag of letters has 4 A's, 3 B's, 3 C's, and 2 D's. How many 12 bit strings can be formed from these letters? (3 points)

$$\binom{12}{4 \quad 3 \quad 3 \quad 2}$$

6) Solve the equation below for $x_1, x_2, x_3$, and $x_4$. Each variable must be a positive integer. Note that a number is positive if it greater than zero. (4 points)

$$x_1 + x_2 + x_3 + x_4 = 16$$

$$x_i' = x_i - 1$$
$$x_1' + x_2' + x_3' + x_4' = 12$$
$$\binom{12+3}{3} = \binom{15}{3} = \binom{15}{12}$$

Note: The question here is badly phrased. Both myself and the students were on autopilot to count the number of solutions, which is what I intended. It should ask "How many solutions are there?" not to solve the equation. If anybody had actually solved the equation as the problem asked, of course they would have received full credit. And maybe even a bonus for actually reading the question…

7) Find the coefficient of $x^7$ in the expression $(2y - x)^{70}$. (4 points)

The entire term is: $\binom{70}{7}(2y)^{63}(-x)^7$, so the coefficient is everything other than the $x^7$:

$$\binom{70}{7} 2^{63}(-1)^7 y^{63}$$

Note that usually the coefficient is just the number part, so full credit was given even if you omitted the $y^{63}$ above.

8) You are dealt 5 cards from a standard deck of 52 cards. What is the probability you do not get a flush? (A flush is when all your cards are the same suit, such as five spades) (4 points)

$$1 - \frac{4 \cdot \binom{13}{5}}{\binom{52}{5}}$$

9) Find the general solution to the recurrence relation given by $a_n = 3a_{n-1} + 10a_{n-2}$. (4 points)

Characteristic equation:

$$x^2 = 3x + 10$$
$$x^2 - 3x - 10 = 0$$
$$(x - 5)(x + 2) = 0$$
$$x = 5, -2$$

General solution:

$$a_n = b \cdot 5^n + c \cdot (-2)^n$$

10) In Dijkstra's algorithm on an arbitrary graph, give and explain a lower bound for the number of times an implementation of the algorithm would have to load any given vertex to the computer's processor. (4 points)

$\Omega(n)$ is the simplest meaningful lower bound on the runtime of the algorithm, because the algorithm in the worst case will have to visit every single vertex.

$\Omega(1)$ is the simplest meaningful lower bound on how many times each vertex must be considered, for the same reasoning.

Either explanation was accepted.

**Use the graph below to solve these problems.**

11) Give an example of a path between A and L. (2 points)

One path is $A - E - I - K - L$. There are multiple answers.

12) Give an example of a cycle through A. (2 points)

One cycle is $A - B - F - E - A$. There are multiple answers.

13) What vertex has the largest degree? (2 points)

J, with degree 6.

14) Run Dijkstra's algorithm on the graph to find the shortest path between A and P. Illustrate your work on the graph itself. (8 points)