Name $\qquad$

Please show all your work and circle your answer when appropriate. You do not need to simplify answers unless the problem specifies to do so.

1) Find an adjacency matrix of the graph shown below. (4 points)


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2) Draw a graph with the incidence matrix below. (4 points)
$\left[\begin{array}{llllll}0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1\end{array}\right]$
3) Find an upper bound, using big-oh, on the number of edges in a graph with $n$ vertices. (4 points)

In the five problems that follow, use $n$ for the number of vertices and $m$ for the number of edges in a graph.
4) Find an upper bound, using big-Oh, on the number of 1 s in an incidence matrix. (3 points)
5) Find an upper bound, using big-Oh, on the number of 1 s in an adjacency matrix. (3 points)
6) Consider the problem "Find the vertex with the largest degree". Find and justify a nonconstant lower bound, using big-Omega, for the time required to solve this problem. (6 points)
7) Consider the problem of "Find the vertex with the largest degree". It can be solved using an adjacency matrix. Specifically we'll iterate through every row of the matrix, and within each row we'll add up all the 1 s in that row. Use this algorithm to find an upper bound, using big-Oh, for the time required to solve this problem. (4 points)
8) Draw the graph below 3 different ways. (Do not make them too similar. I want to make sure you understand what information in a graph representation is important and what can be changed) (3 points)

9) The graph below is planar. However, the drawing of it is not a planar representation. Draw a planar representation of this graph. (4 points)

10) Color the graph below with three colors. (Recall that adjacent vertices cannot share a color) (4 points)

11) Draw the complete bipartite graph $K_{3,2}$. (3 points)
12) Find a minimal spanning tree of the graph shown below. (4 points)

13) Use a breadth-first algorithm to find a spanning tree of the graph shown below. Start at the vertex labelled "S". (4 points)


