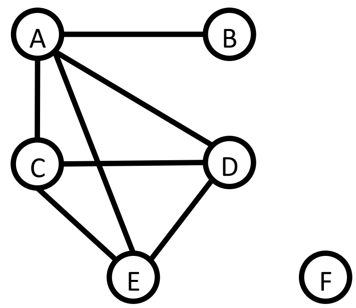
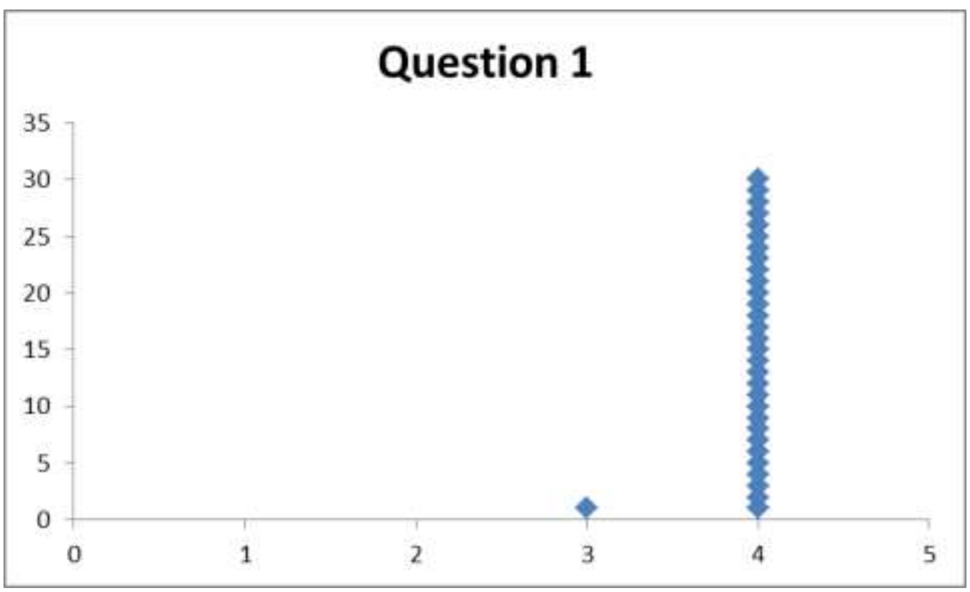
Name __Solutions___________________________________________ Discrete II, Big Quiz 2, Spring 2017

**Please show all your work and circle your answer when appropriate. You do not need to simplify answers unless the problem specifies to do so.**

1) Find an adjacency matrix of the graph shown below. (4 points)

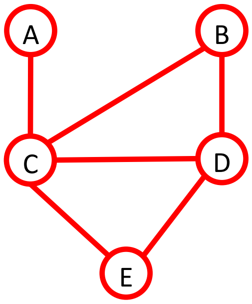
$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

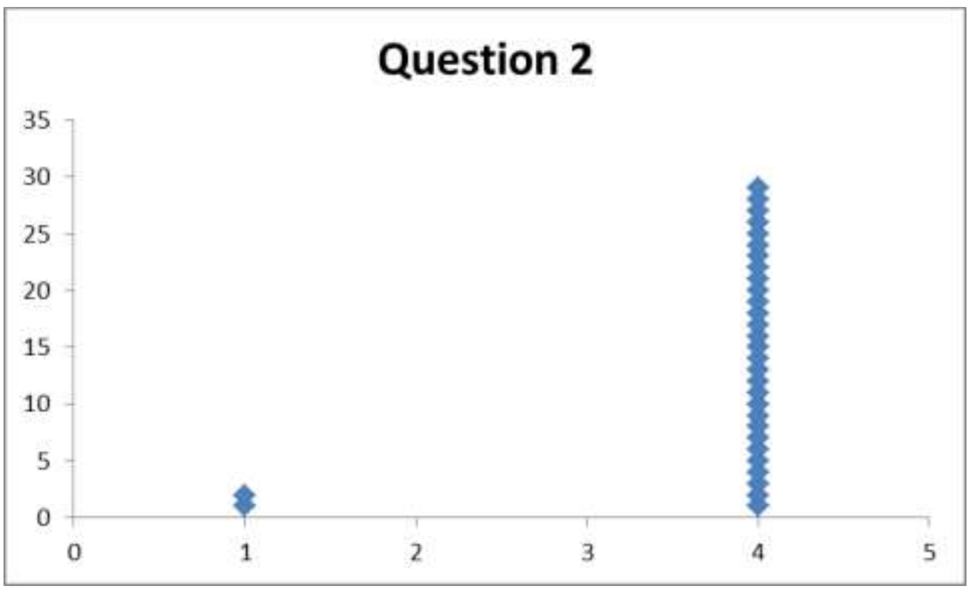


2) Draw a graph with the incidence matrix below. (4 points)

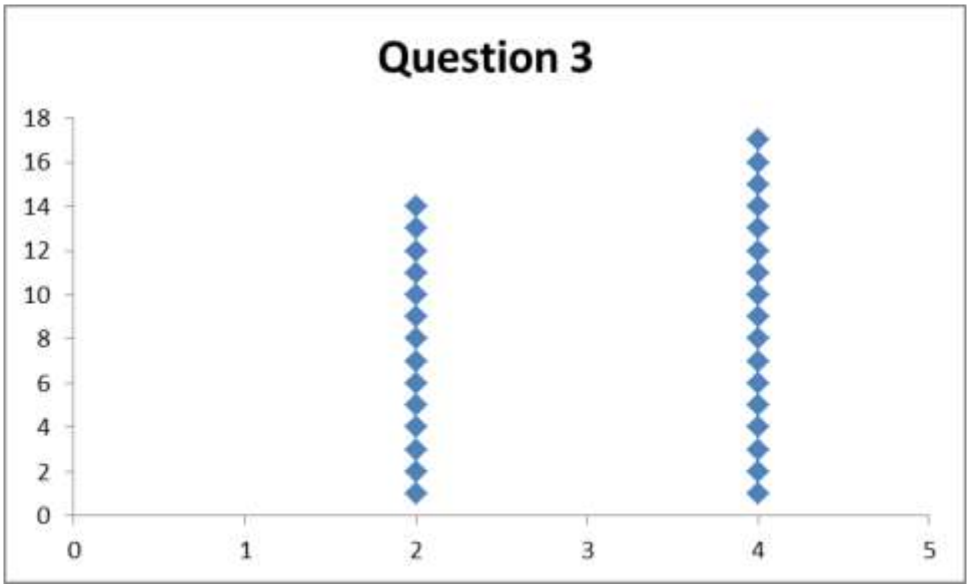
$$\begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$



3) Find an upper bound, using big-oh, on the number of edges in a graph with $n$ vertices. (4 points)

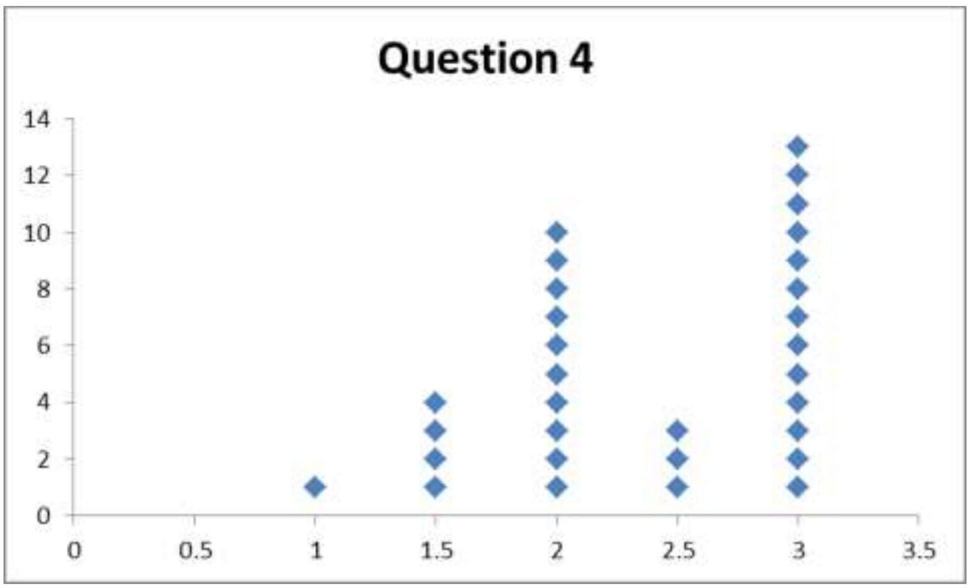
When *graph* is used without additional qualifiers, it means a finite simple graph. Hence the complete graph $K_n$ has as many edges as possible. How many edges is that? Between every two vertices. How many pairs of vertices are there? $\binom{n}{2} = \frac{n(n+1)}{2}$ which is $O(n^2)$



In the **five** problems that follow, use $n$ for the number of vertices and $m$ for the number of edges in a graph.

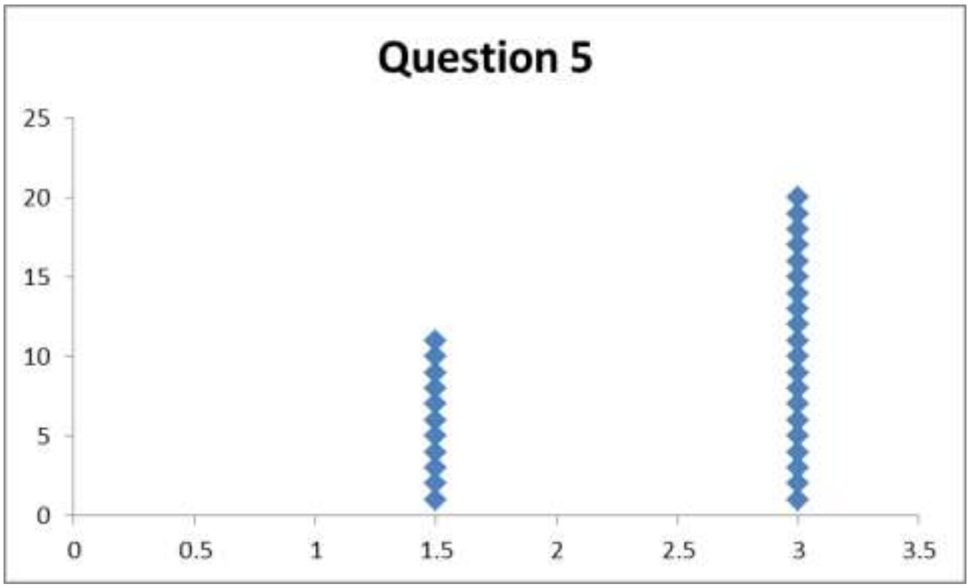
4) Find an upper bound, using big-Oh, on the number of 1s in an incidence matrix. (3 points)

Every edge (column) has exactly two 1s, so that is exactly $2m$. Under big-Oh this is $O(m)$.



5) Find an upper bound, using big-Oh, on the number of 1s in an adjacency matrix. (3 points)

The complete graph has a 1 in every position off the diagonal. Similar to question #3, this is $O(n^2)$



6) Consider the problem "Find the vertex with the largest degree". Find and justify a nonconstant lower bound, using big-Omega, for the time required to solve this problem. (6 points)

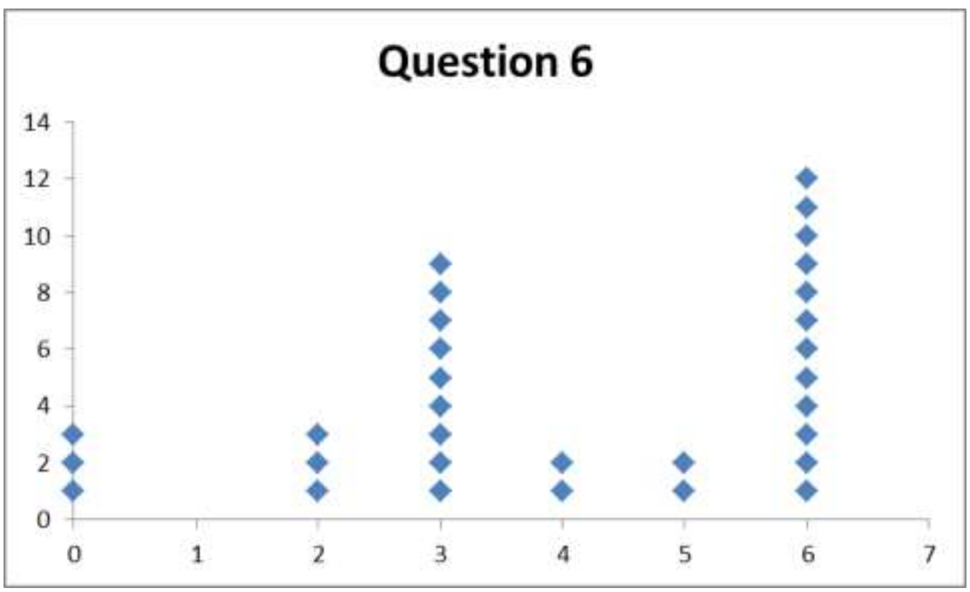
The simplest bound and explanation is probably:

$\Omega(n)$ because we need to look at every vertex.

You can also note that in the worst case you'll also need to consider every edge, which gives a lower bound of $\Omega(m)$.

Together that gives us $\Omega(n+m)$.

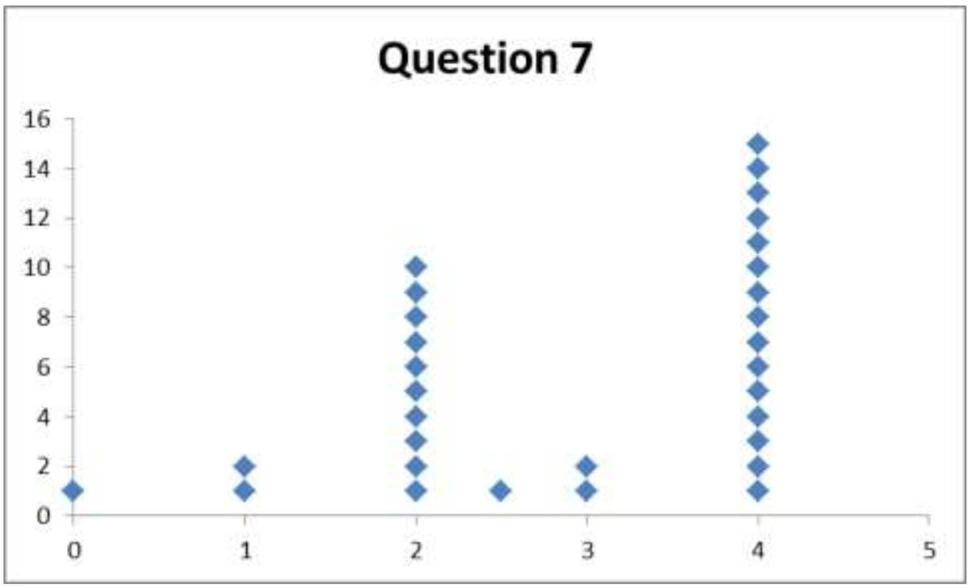
Also of interesting note is that this is the lower bound on *any* graph algorithm: because in order to input a graph to an algorithm we must, somehow, store every vertex and every edge.



7) Consider the problem of "Find the vertex with the largest degree". It can be solved using an adjacency matrix. Specifically we'll iterate through every row of the matrix, and within each row we'll add up all the 1s in that row. Use this algorithm to find an upper bound, using big-Oh, for the time required to solve this problem. (4 points)

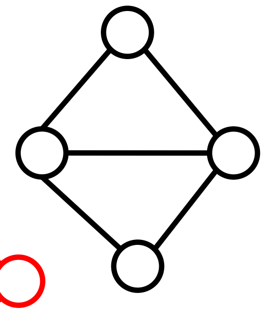
This algorithm is essentially two loops of length $n$. One going through the rows for each vertex, and one going through the columns to add up the 1s. Note that a different algorithm on a different data structure could computer this faster, but that is not what is described here.

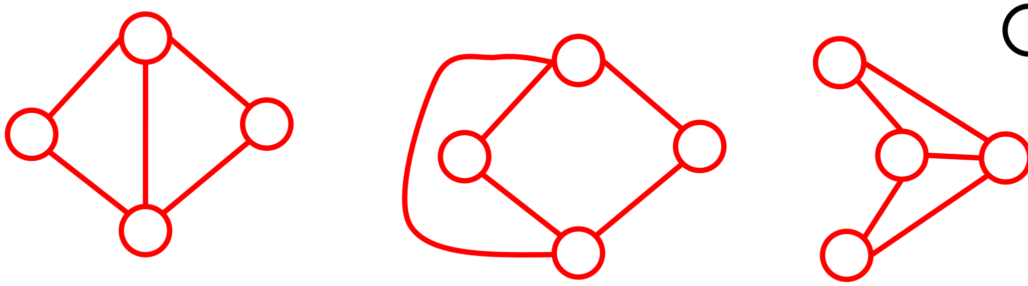
$$O(n^2)$$

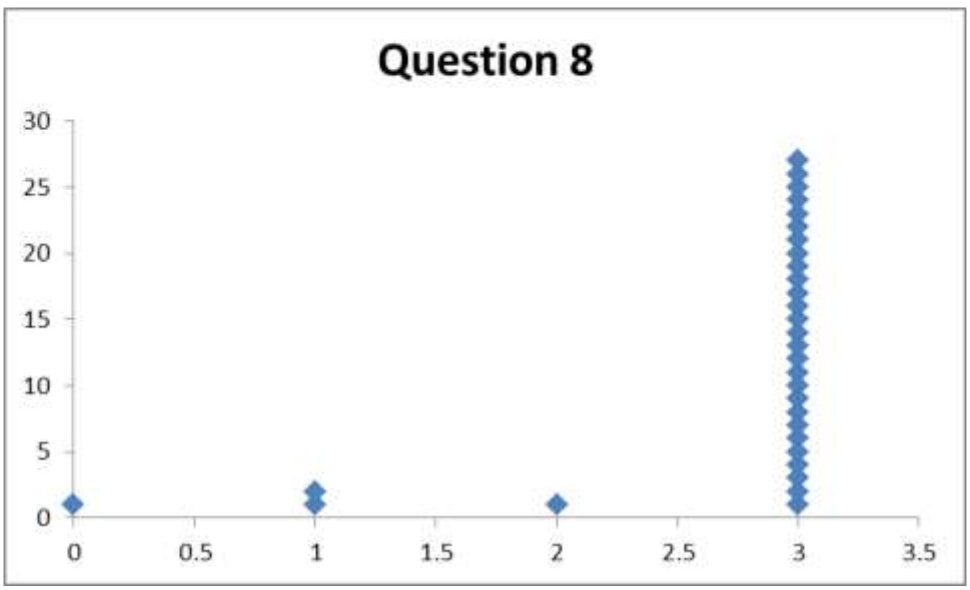


8) Draw the graph below 3 different ways. (Do not make them too similar. I want to make sure you understand what information in a graph representation is important and what can be changed) (3 points)

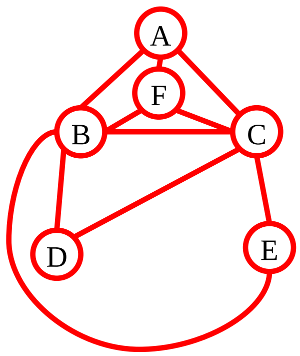
There are many answers, here are a few:

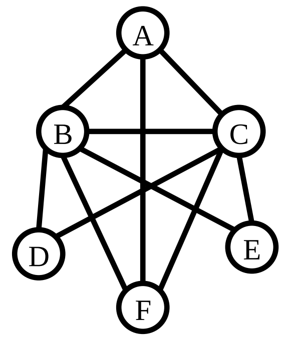


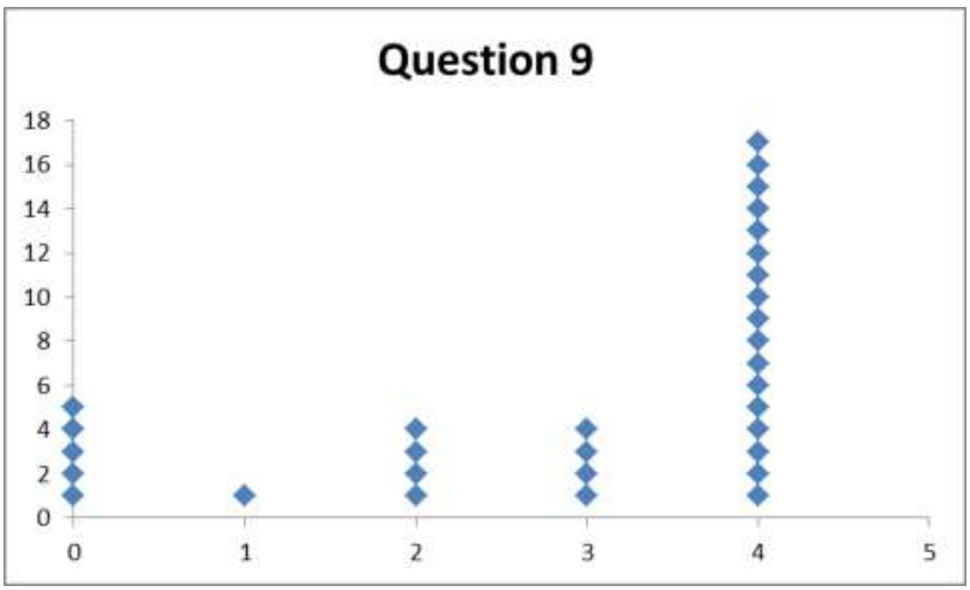




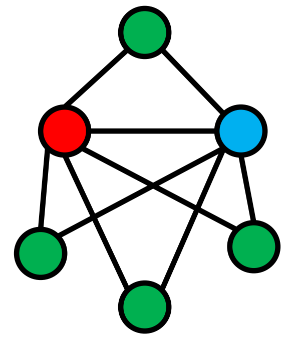
9) The graph below is planar. However, the drawing of it is not a planar representation. Draw a planar representation of this graph. (4 points)

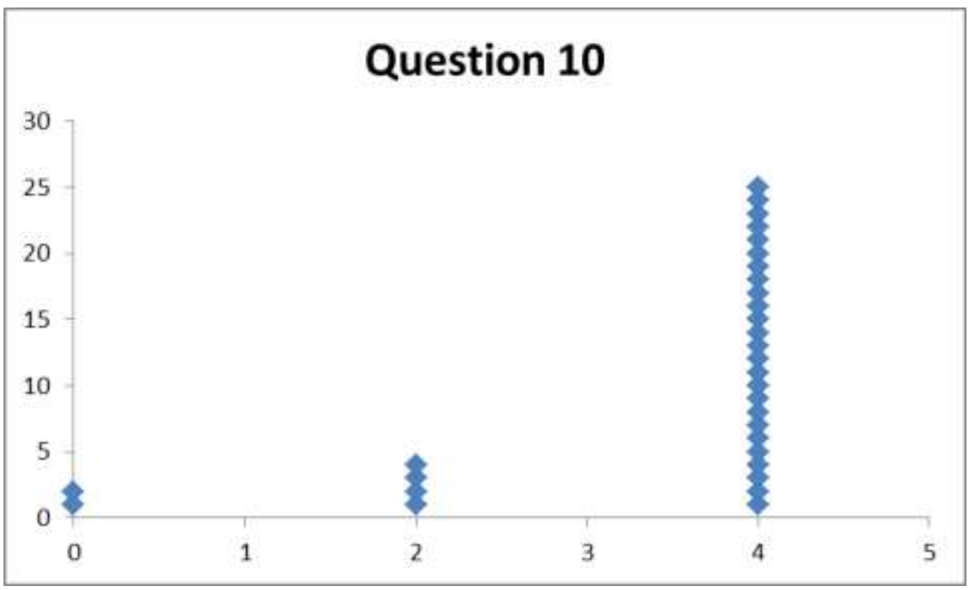


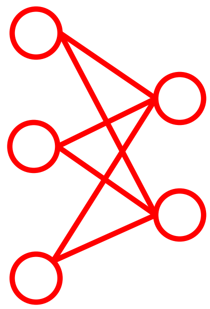



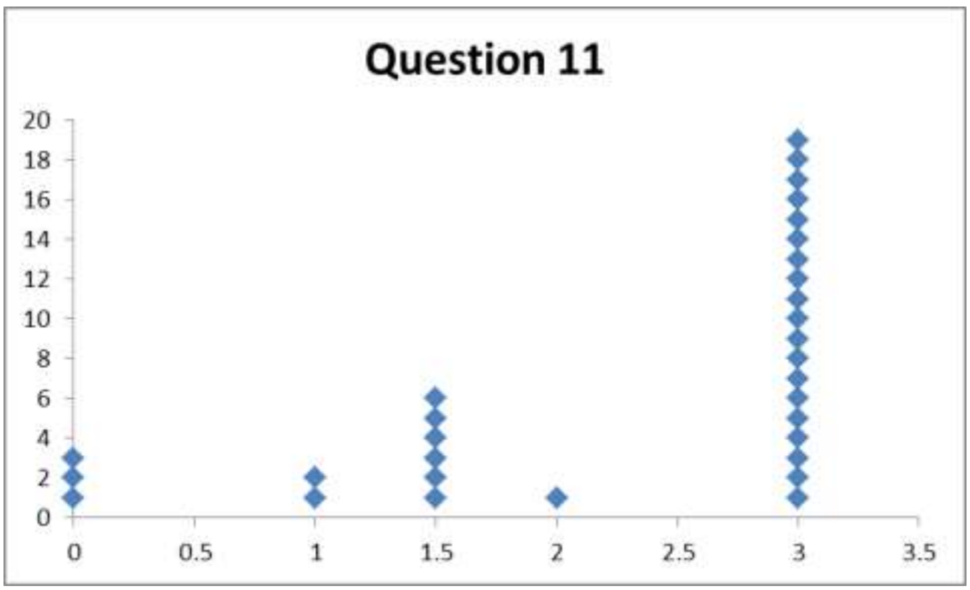
10) Color the graph below with three colors. (Recall that adjacent vertices cannot share a color) (4 points)





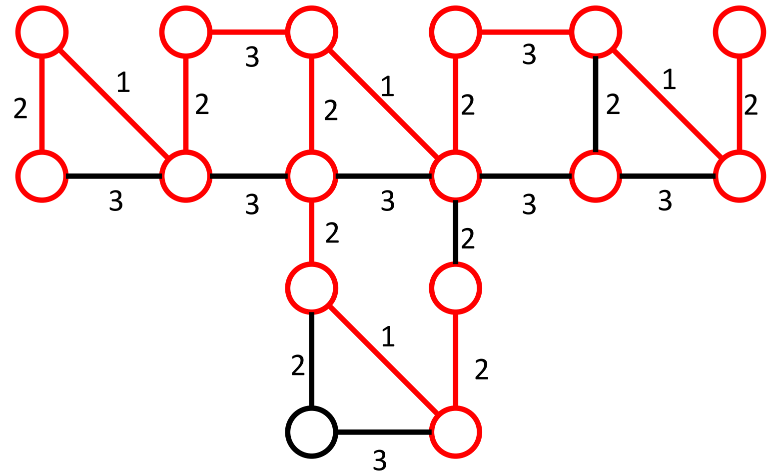
11) Draw the complete bipartite graph $K_{3,2}$. (3 points)

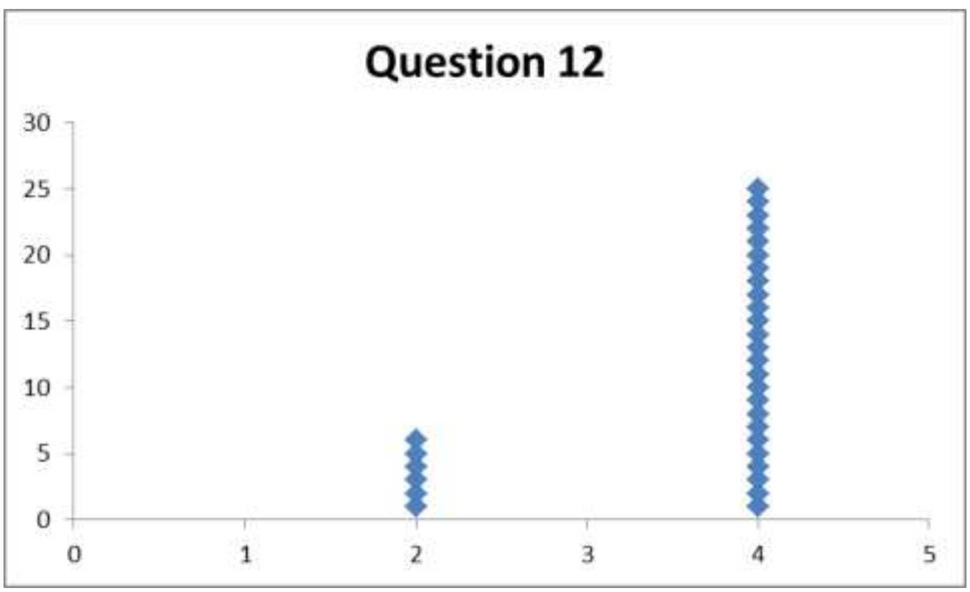




12) Find a minimal spanning tree of the graph shown below. (4 points)

There are multiple answers.



13) Use a breadth-first algorithm to find a spanning tree of the graph shown below. Start at the vertex labelled "S". (4 points)

There are multiple answers.