Name ___Solutions____Discrete II, Spring 2017, Quiz 3

1) A sequence is given by $T_n = 6T_{n-2} - T_{n-1}$, and it is known that $T_0 = 8$. Find four solutions to this recurrence relation.

Characteristic equation:

$$x^2 = 6 - x$$

$$x^2 + x - 6 = 0$$
$$(x-2)(x+3) = 0$$
$$x = 2; x = -3$$

Two solutions to the recurrence relation without the initial condition:

$$S_1 = 2^n, S_2 = (-3)^n$$

General solution:

$$T_n = a2^n + b(-3)^n$$

Initial condition:

$$8 = a + b$$

From here we can get as many solutions as you like, such as:

$$T_n = 8 \cdot 2^n$$
$$T_n = 7 \cdot 2^n + (-3)^n$$
$$T_n = 6 \cdot 2^n + 2 \cdot (-3)^n$$
$$T_n = 5 \cdot 2^n + 3 \cdot (-3)^n$$

2) A sequence is given by $T_n = 10T_{\left\lfloor \frac{n}{3} \right\rfloor} + n^2$. Find an asymptotic expression for T_n using big-oh.

Using the main recurrence theorem (part 1), we compare 5 to 3^2 and see that the third part is what we need:

$$T_n \text{ is } O\left(n^{\log_3(10)}\right)$$