Name ____ Quiz 3

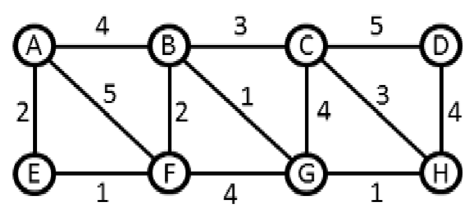
1) On the graph below, use Dijkstra's algorithm to find a shortest path between vertex A and vertex H.

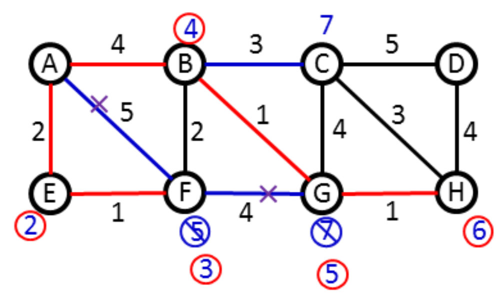
Be sure to clearly illustrate how the algorithm works.

You need to illustrate how the algorithm works. There are multiple methods to do this. The method I illustrated in class is below, where blue represents processing nodes, and red represents known shortest paths.

If you traced the algorithm correctly, your work should reflect the following key features:

- The shortest path is A-F-B-G-H with a total length of 6. (15 points)
- The distance to both F and G was 5 and 7, but later reduced (10 points)
- The shortest path to vertex F is A-E-F with a total length of 3 (10 points)
- The shortest path to vertex D is not known (10 points)
- The edges A-F and F-G were at one point used, but better routes were found (5 points)





2) Assume we have an implementation of Dijkstra's algorithm that requires $\Theta(n^2)$ time to run through the entire graph. We will use this algorithm to solve the "all pairs shortest path" problem by iterating through each vertex and running Dijkstra's algorithm from that vertex. In doing so, we will find the shortest path between every pair of vertices.

How long will it take to solve the "all pairs shortest path" problem using this method?

$\Theta(n^3)$ because we are iterating a $\Theta(n^2)$ algorithm over n vertices.

3) In the previous question you found a runtime for a particular algorithm for solving the "all pairs shortest path". Find and justify a lower bound on the time required to solve this problem.
(For full credit your lower bound should be nontrivial)

$\Omega(n)$ – we must consider every vertex. (24 points)
$\Omega(m)$ – we must consider every edge. (32 points)
$\Omega(n^2)$ – we must consider every pair of vertices, of which there are $\binom{n}{2}$ pairs. (50 points)
$\Omega(\text{something})$ – 12 points

1/3rd credit if no justification.
1/2 credit if incorrect justification.
2/3rd credit if correct but insufficient justification

**Easiest way to get no credit: Present $O(\text{something})$ as a lower bound. Big-oh creates asymptotic upper bounds, not lower bounds.
**Easiest way to get an incorrect justification: Present $\Omega(\text{something})$ as a best-case scenario. All give of the asymptotics measures give worst-case performance, including big-omega.