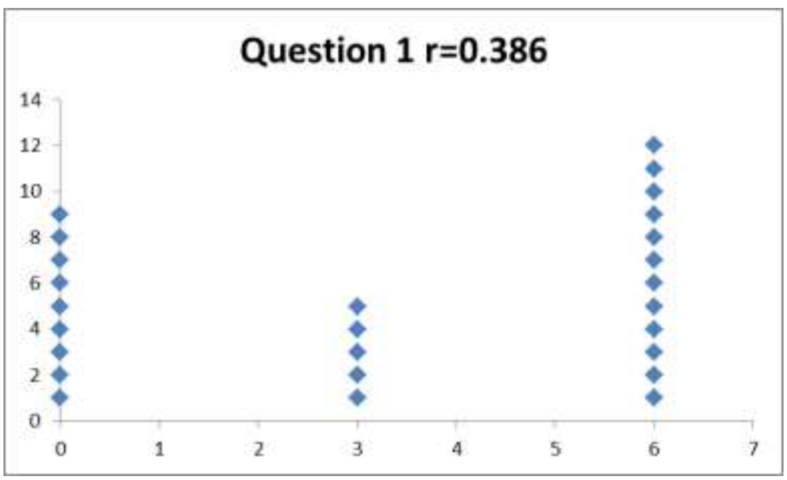
Name ____Test 1, Discrete II, Spring 2018

Directions: Complete each problem. Do not simplify answers, except on the very first problem. If it's not obvious what your final answer is, circle it.

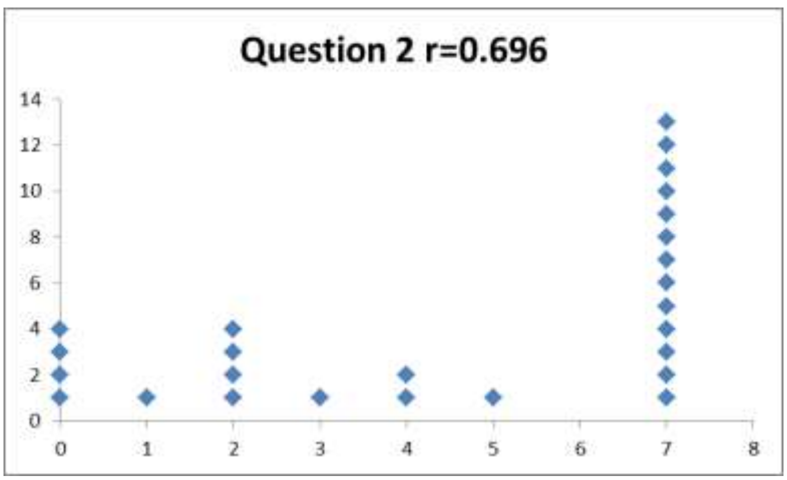
1) Calculate $\binom{6}{4}$.
(6 points)

$$\binom{6}{4} = \frac{6!}{4!\,2!} = \frac{6\cdot5\cdot4\cdot3}{4\cdot3\cdot2\cdot1} = \frac{6\cdot5}{2\cdot1} = 15$$



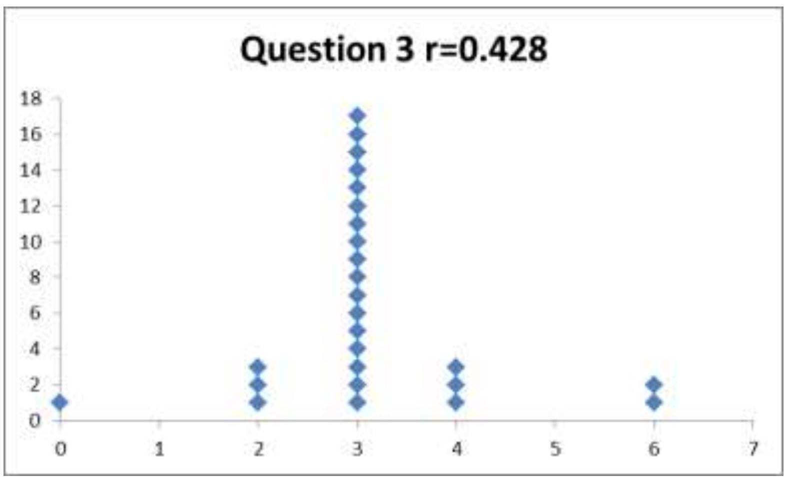
2) Given the set $\{1,4,5,6,7\}$, find the first seven permutations in lexicographic ordering.
(7 points)

1 4 5 6 7
1 4 5 7 6
1 4 6 5 7
1 4 6 7 5
1 4 7 5 6
1 4 7 6 5
1 5 4 6 7



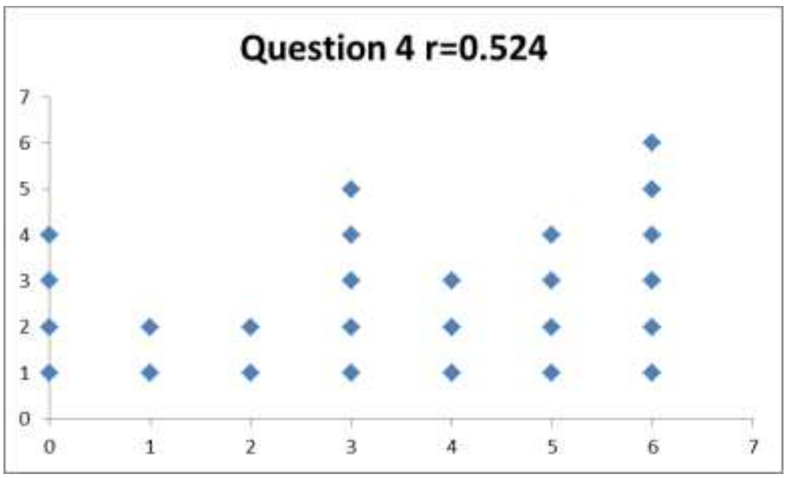
3) Suppose there are 12 roads from Conway to Little Rock and 5 roads from Little Rock to Hot Springs. How many round trips are there from Conway–Little Rock–Hot Springs–Little Rock–Conway that do not reverse the original route from Conway to Hot Springs?
(6 points)

$$12 \cdot 5 \cdot (5 \cdot 12 - 1)$$



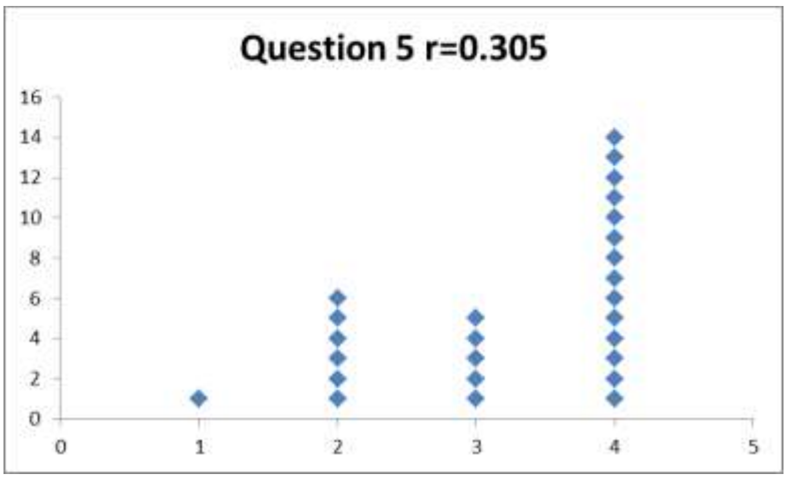
4) At a certain age group, 10% of the population is infected by a disease. A test for infection is 99% accurate when you're infected, but only 95% accurate when you're not infected. If you take the test and it says you are infected, what is the probability that you are actually infected?
(6 points)

$$P(Infected|TestP) = \frac{P(Infected \cap TestP)}{P(TestP)} = \frac{0.1 \cdot 0.99}{0.1 \cdot 0.99 + 0.9 \cdot 0.05}$$



5) How many numbers are between 71 and 293, inclusive?
(4 points)

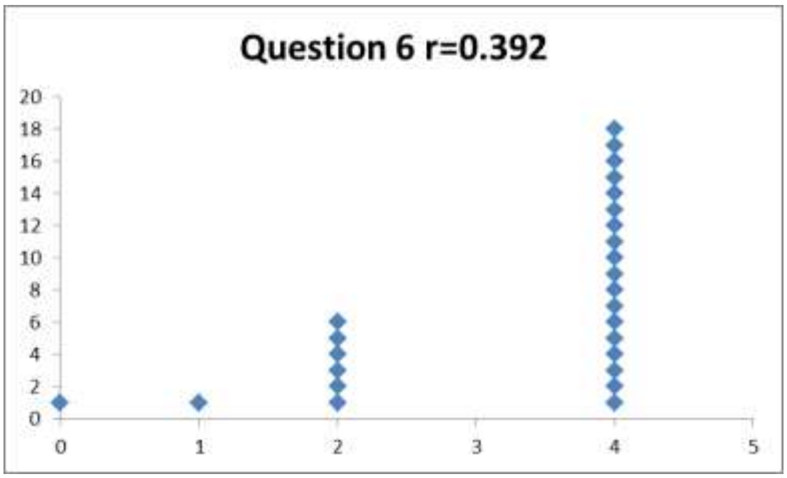
$$293 - 71 + 1$$



6) Determine how many strings can be formed by ordering the ten letters ABCDEFGHIJ that contain the substrings AB and EC?

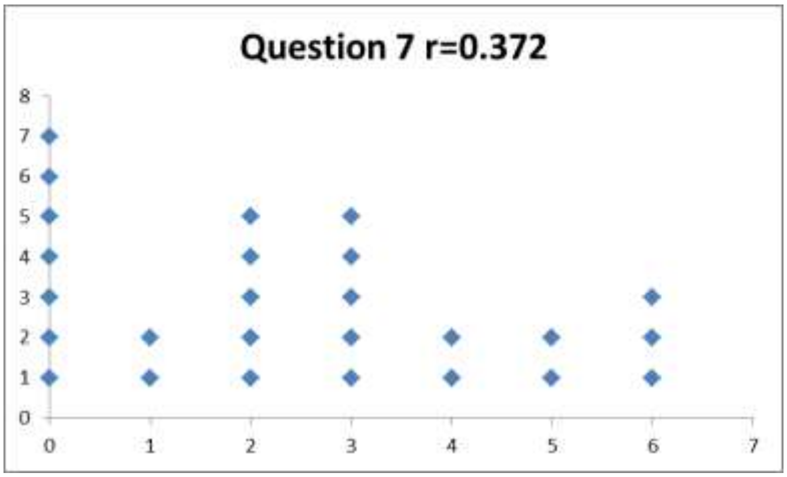
(4 points)

8!



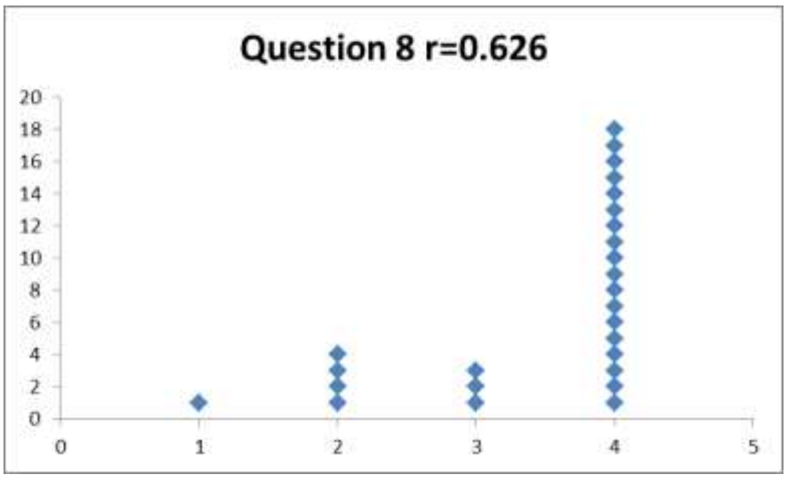
7) Find the coefficient of $w^2x^3y^2z^5$ in $(2w + x + 3y + z)^{12}$.
(6 points)

$$\binom{12}{2\ 3\ 2\ 5}2^2 3^2$$



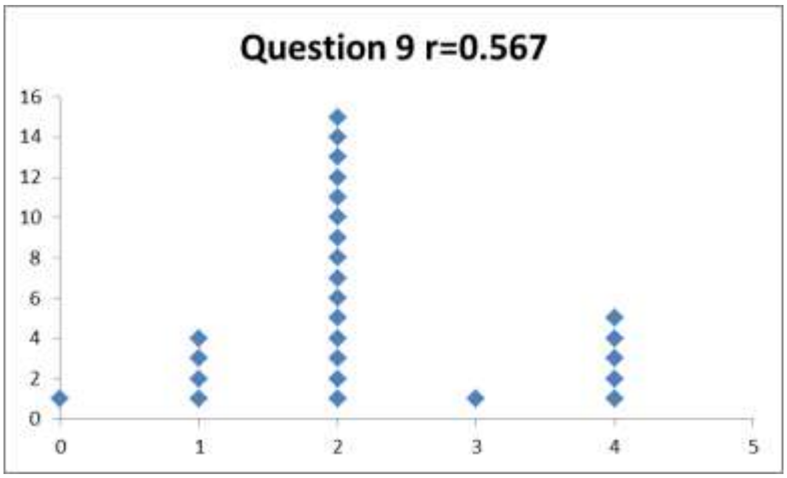
8) A coin is tossed 42 times. How many different outcomes are possible if each outcome is to be written down sequentially?
(4points)

2^{42}



9) 42 coins are tossed. How many different outcomes are possible if the coins that land heads will be donated to the democrats and the coins that land tails will be donated to the republicans?
(4 points)

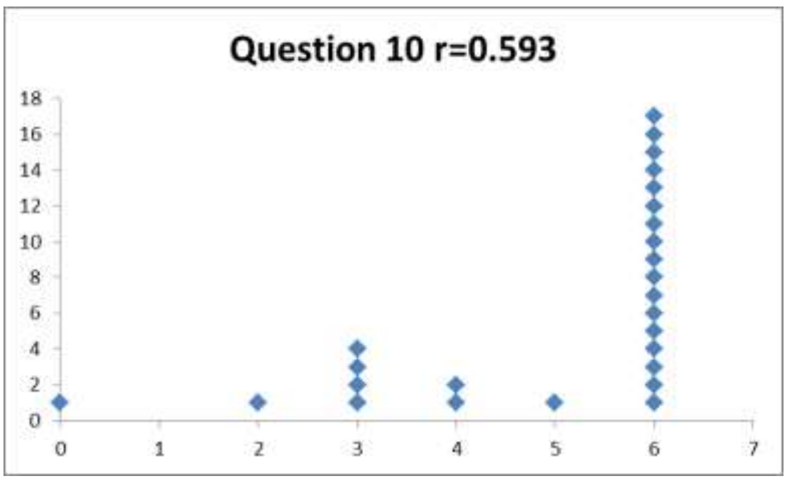
43



10) Find the number of positive integer solutions to $x_1 + x_2 + x_3 + x_4 + x_5 = 27$ subject to the condition that $x_3 \geq 4$.
(6 points)

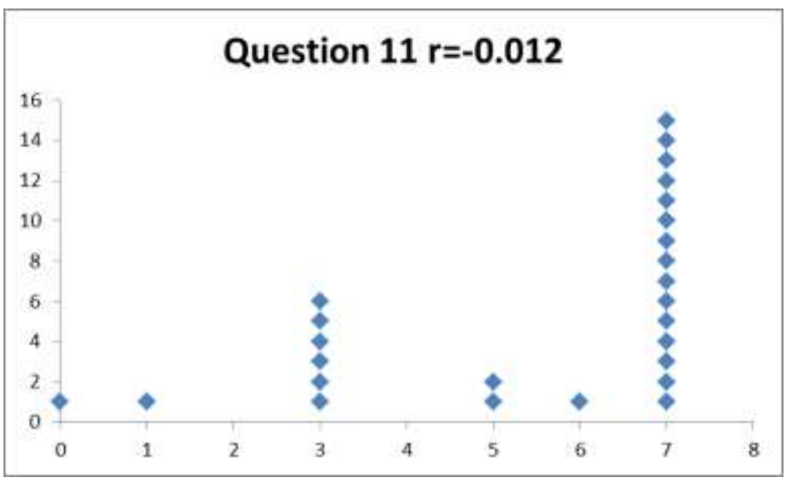
$$x_1 + x_2 + y_3 + x_4 + x_5 = 23$$

$$\binom{23+4}{4}$$



11) Given the set $\{1,4,5,6,7\}$, find the first seven 3-combinations in lexicographic ordering.
(7 points)

1 4 5
1 4 6
1 4 7
1 5 6
1 5 7
1 6 7
4 5 6



12) How many integers between 1 and 1,000,000 have a sum of the digits equal to 15?
(8 points)

$$n = abcdef$$

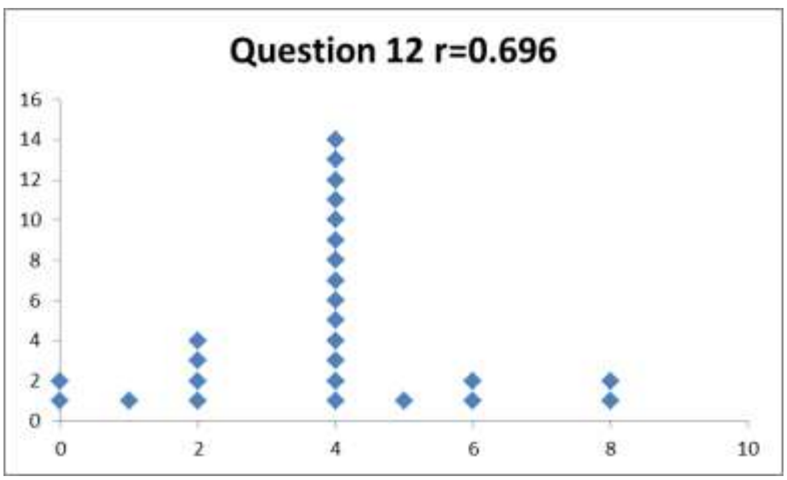
$$a + b + c + d + e + f = 15$$

$$\binom{15+5}{5}$$

$$A + b + c + d + e + f = 5$$

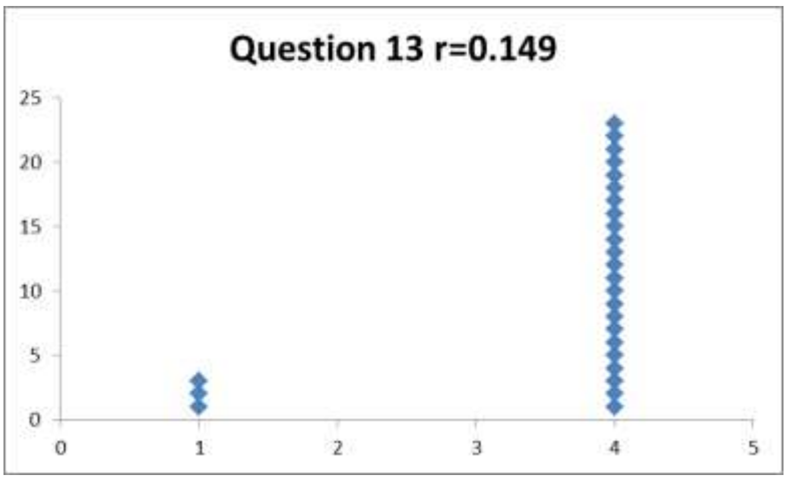
$$\binom{5+5}{5}$$

$$\boxed{\binom{15+5}{5} - 6\binom{5+5}{5}}$$



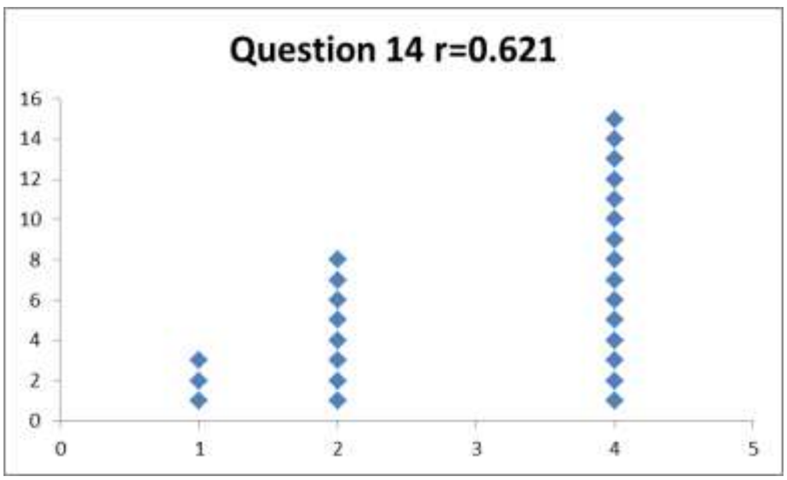
13) A fair six-sided die is rolled. What is the probability of an even number?
(4 points)

$$\frac{1}{2}$$



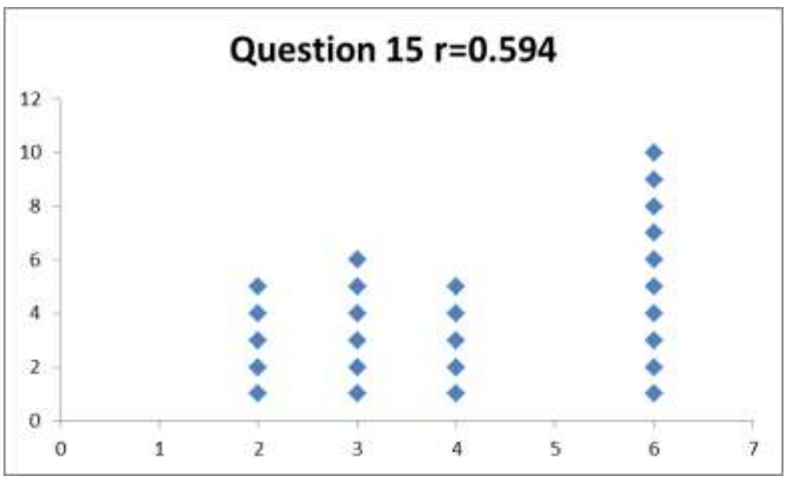
14) A weighted die is rolled. What is the probability of an even number? It is weighted so that 2, 4, and 6 are equality likely to appear. 1, 3, and 5 are also equally likely to appear, but 1 is three times as likely as 2 to appear.
(4 points)

$$\frac{1}{4}$$



15) Among 100 people, 10 of them have a certain disease. If you select 6 people at random, what is the probability that none of them are infected?
(6 points)

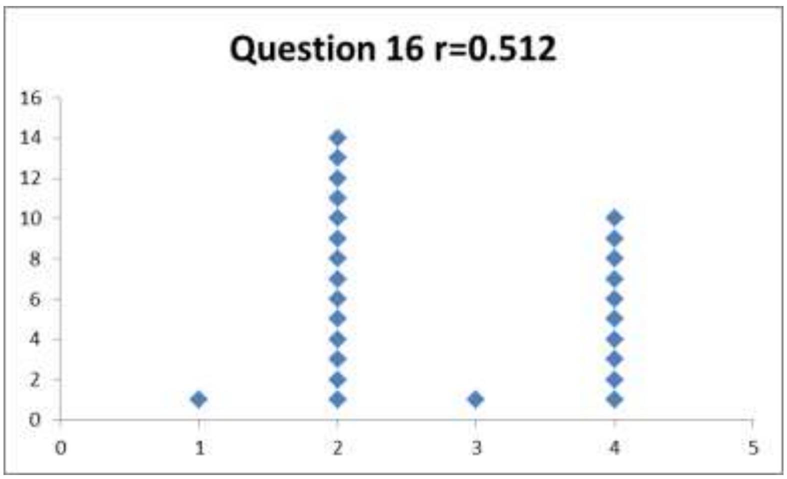
$$\frac{\binom{90}{6}}{\binom{100}{6}}$$



16) A person invests \$2000 with a 4% annual rate of return. Let A_n represent the amount in the account at the end of n years. Find an explicit (closed form) formula for A_n

(4 points)

$$A_n = 2000 \cdot 1.04^n$$



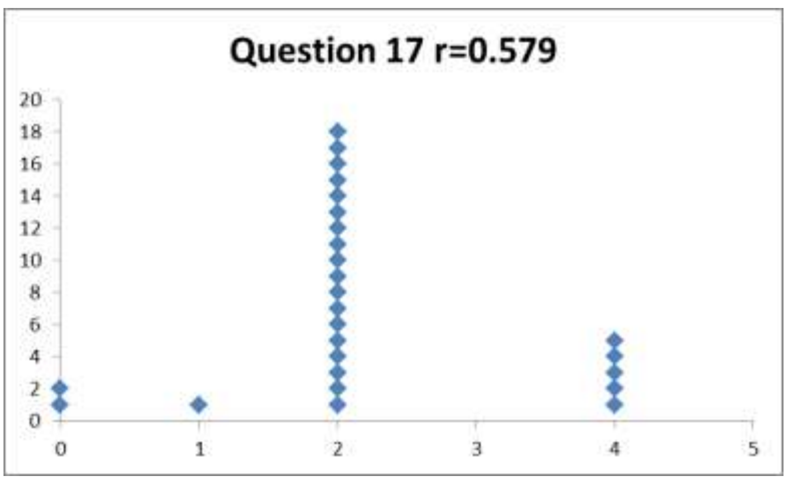
17) A person invests \$2000 annually with a 4% annual rate of return. Let A_n represent the amount in the account at the end of n years. Find a recurrence formula for A_n
(4 points)

$$A_n = 1.04 \cdot A_{n-1} + 2000$$

OR

$$A_n = 1.04 \cdot (A_{n-1} + 2000)$$

These are not the same, but are correct for different interpretations of when you invest the 2000. (Namely at the start or end of the year)



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18) Let $a_n = 6a_{n-1} - 8a_{n-2}$ with the initial conditions $a_0 = 0$ and $a_1 = 10$. Solve this recurrence relation to find an explicit formula for a_n.
(10 points)

$$x^2 = 6x - 8$$
$$x^2 - 6x + 8 = 0$$
$$(x-4)(x-2) = 0$$

$$x = 2, 4$$

$$s_1 = 2^n; s_2 = 4^n$$

$$a_n = b2^n + c4^n$$

$a_0 = 0: b + c = 0$

$$b = -c$$

$a_1 = 10: 2b + 4c = 10$

$$\therefore 2b - 4b = 10$$
$$b = -5$$

$$c = 5$$

$$a_b = -5 \cdot 2^n + 5 \cdot 4^n$$