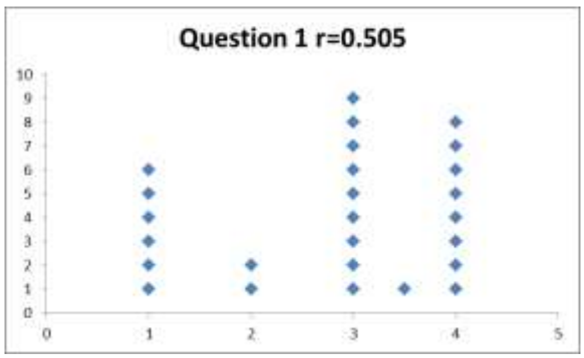
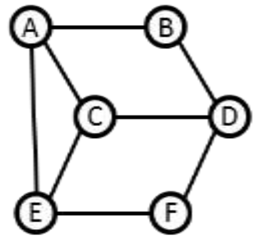
Name ____Test 2, Discrete II, Spring 2018

1) A graph has n vertices and m edges. A particular graph algorithm is going to iterate through every vertex, and run a subroutine that requires $\theta(m)$ runtime. What is the asymptotic runtime of the whole algorithm?
(4 points)

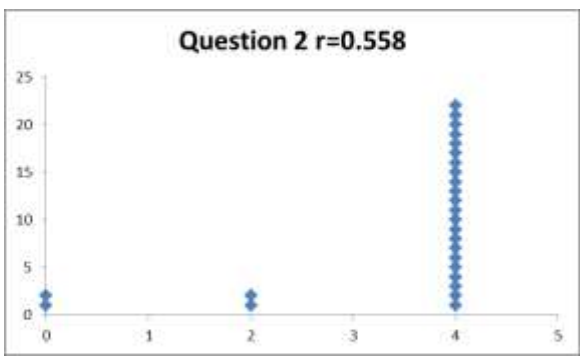
For each of the n vertices, it performs $\theta(m)$ work. Hence it requires $\theta(nm)$ runtime.



2) Construct the adjacency matrix for the graph below.
(4 points)



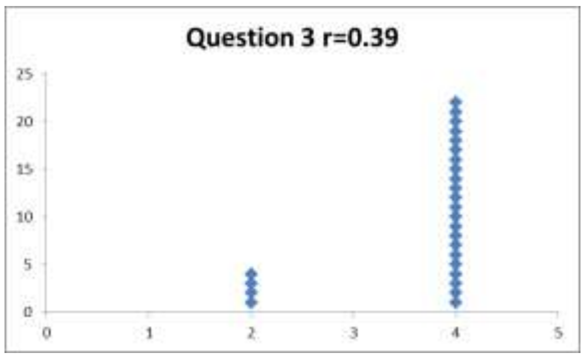
$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$



3) Convert the decimal number 456 to base 5.
(4 points)

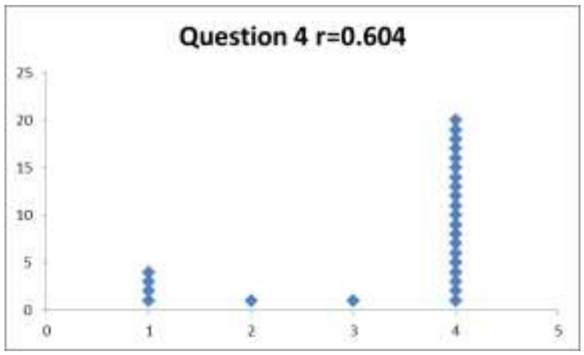
$$456 \div 5 = 91\ R1$$
$$91 \div 5 = 18\ R1$$
$$18 \div 5 = 3\ R3$$
$$3 \div 5 = 0\ R3$$

$$(456)_{10} = (3311)_5$$



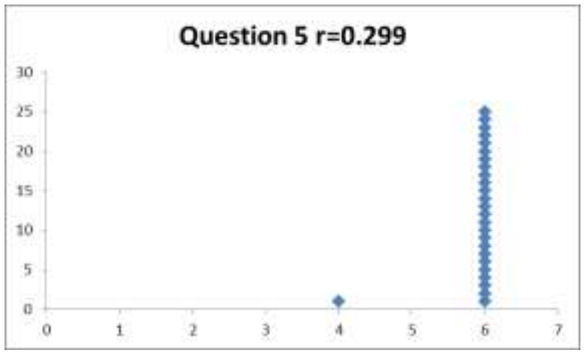
4) Convert the base five number $(123)_5$ to base 10.
(4 points)

$$(123)_5 = 1 \cdot 25 + 2 \cdot 5 + 3 \cdot 1 = 38$$



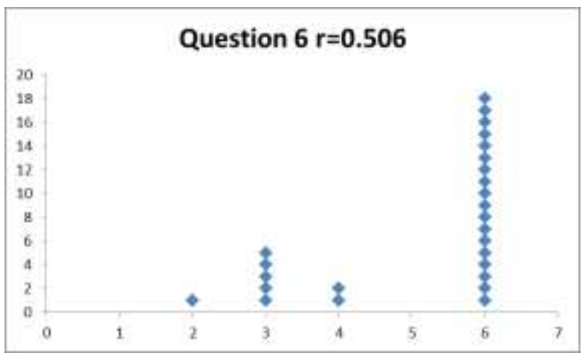
5) Add $(234)_7$ and $(15)_7$ in base 7.
(6 points)

$$\begin{array}{r} 234 \\ +\ \ 15 \\ \hline 252 \end{array}$$



6) Multiply $(234)_7$ and $(14)_7$ in base 7.
(6 points)

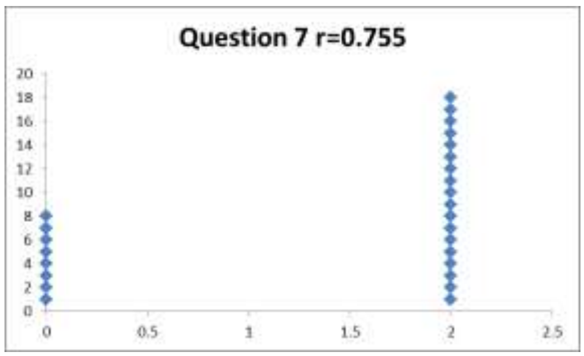
```
  234
 x 14
 1302
+2340
 3642
```



7) Multiple Choice: Do you know what “one’s complement” and “two’s complement” are? Choose the answer that best describes them.

(A) They are methods for calculating the big-Oh runtime of an algorithm
(B) They are methods for representing integers on computers
(C) They are methods for representing graphs on computers
(D) They are methods for negating quantified logical statements

(2 points)

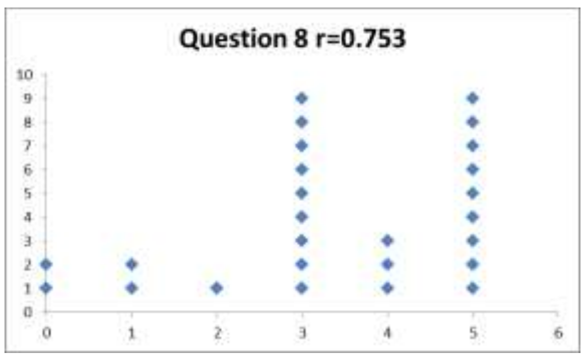


8) It is known that $f(n) = 3n^3 - 2n^2 + 6n$ is $O(n^3)$. Verify this mathematically.
(6 points)

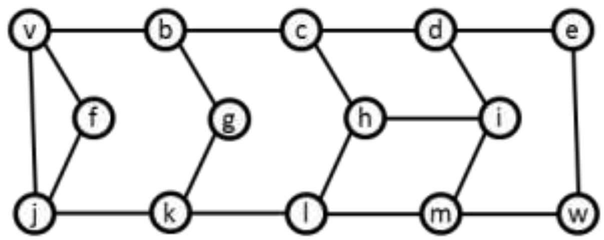
$$3n^3 - 2n^2 + 6n \leq 3n^3 + 6n \leq 3n^3 + 6n^3 = 9n^3$$

Hence $3n^3 - 2n^2 + 6n$ is $O(n^3)$

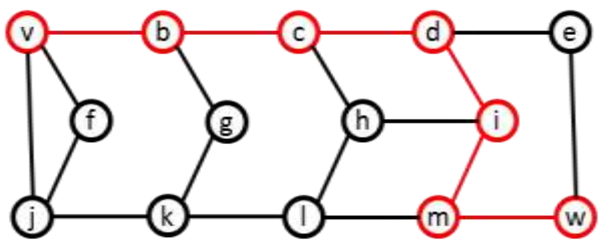
Note that $n \geq 1$ and we need to find an upper bound. So that negative term, $2n^2$ can be removed to make the expression larger. We *cannot* replace it with $2n^3$ because that would make the expression smaller, not larger.

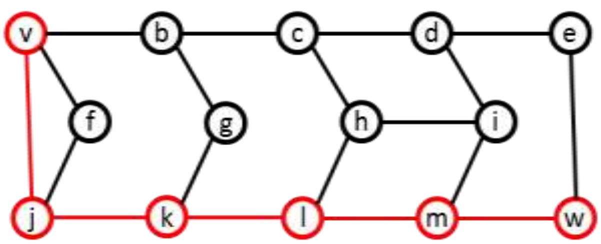


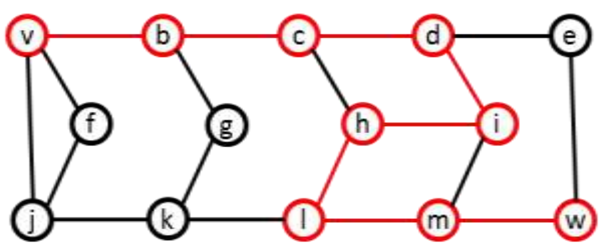
9) On the graph below, find and identify a path from vertex v to w that passes through only vertices of degree 3.
(6 points)

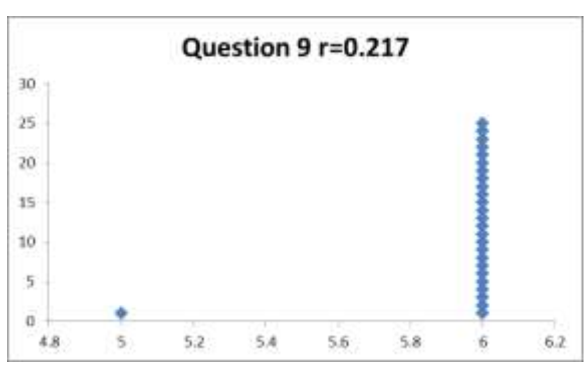


There are several such paths. A few examples are below.

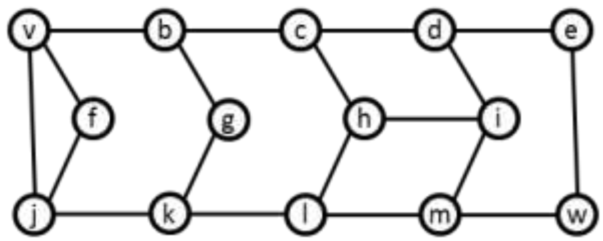




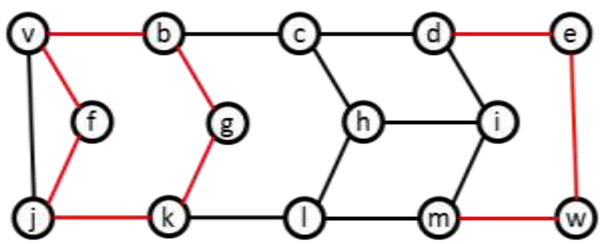


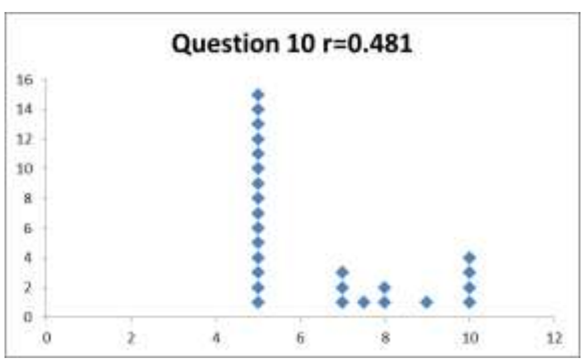


10) On the graph below find a Hamiltonian Cycle or explain/illustrate why one does not exist.
(10 points)

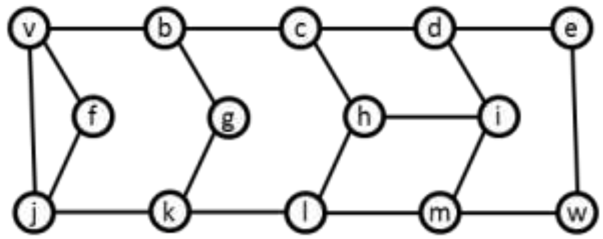


This does not have a Hamiltonian cyclone. You must mathematically justify why there is not one. The easiest way is to consider that all edges incident to a vertex of degree 2 must be included. Removing irrelevant edges and iterating this process we see that we end up with a cycle too early and thus cannot get a Hamiltonian cycle.

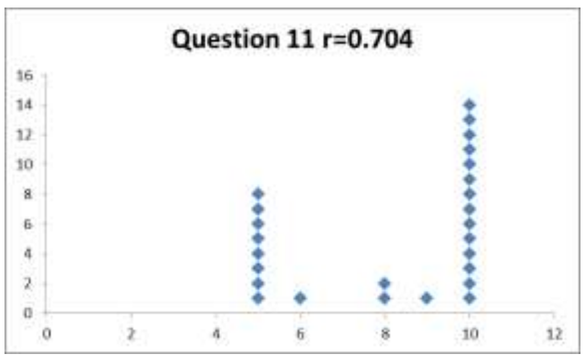




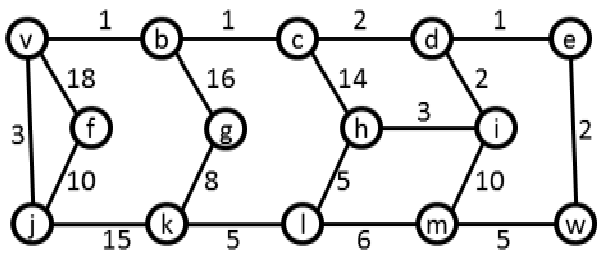
11) On the graph below find an Eulerian Cycle or explain/illustrate why one does not exist.
(10 points)



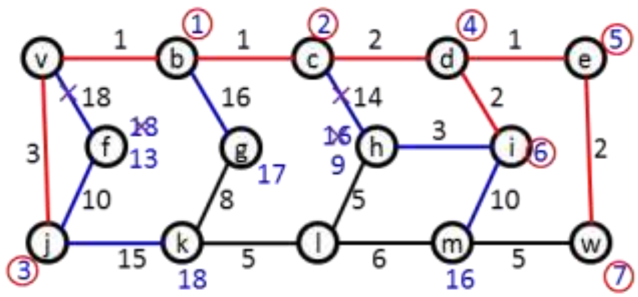
This cannot have an Eulerian cycle because there are vertices of odd degree.



12) On the graph below run Dijkstra's Algorithm to find a shortest path from "v" to "w" or explain/illustrate why one does not exist
(10 points)

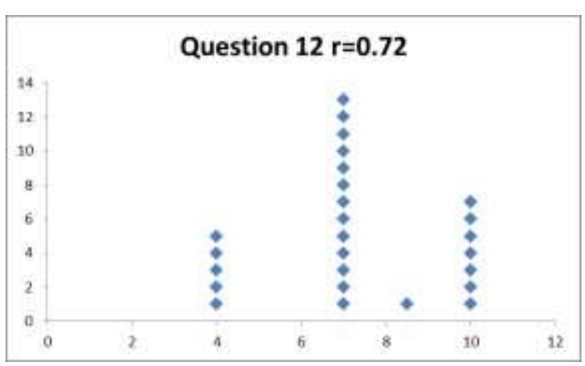


In the illustration here blue represents investigated vertices and paths, red represents optimal distances and paths.



If you traced the algorithm correctly, your work should reflect the following key features:

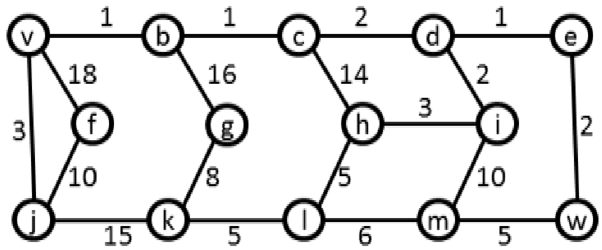
- The distance to vertex f was originally 18 from v, and then later 13 from j.
- The distance to l is unknown.
- The distance to w is 7 via v-b-c-d-e-w



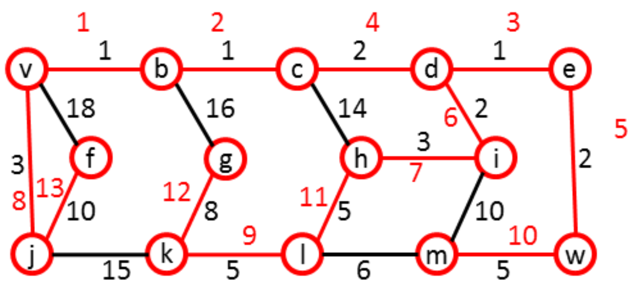
13) Find a minimal spanning tree of the graph below. Label the edges in the order in which you add them to the tree. (1, 2, 3, 4, 5, etc).

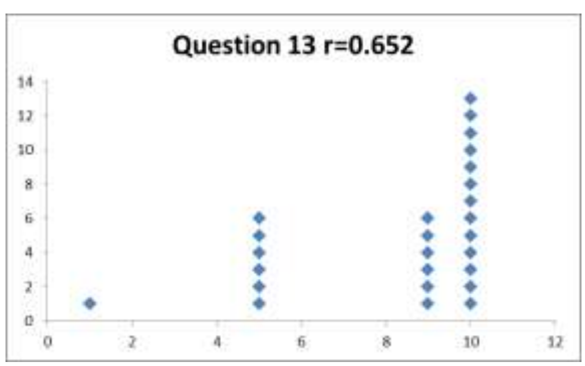
Which algorithm did you use? Prim's Kruskal's (Circle one)

(10 points)



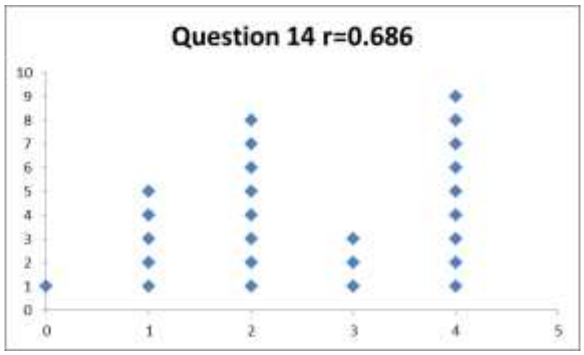
Kruskal's is probably the easier choice, but Prim's isn't too much more difficult.





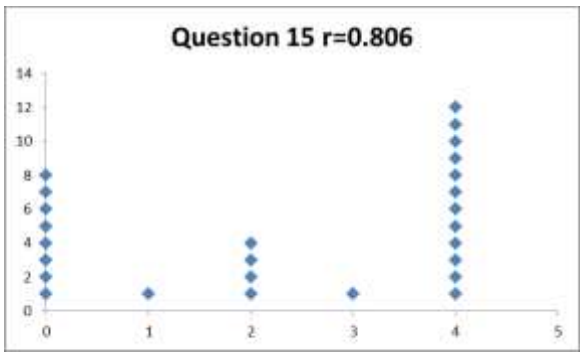
14) If a binary tree has k levels, find an asymptotic upper bound on the number of vertices in the tree.
(4 points)

$O(2^k)$



15) If a binary tree has k levels, find an asymptotic lower bound on the number of vertices in the tree.
(4 points)

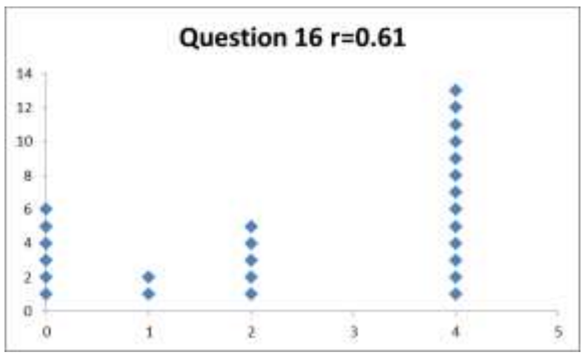
$\Omega(k)$



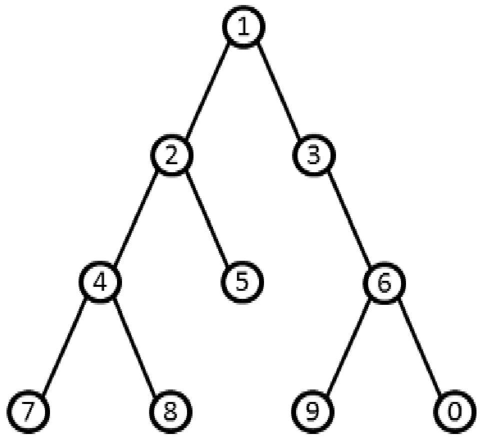
16) If a binary tree has k levels, find an asymptotic upper bound on the number of roots in the tree.
(4 points)

$$O(1)$$

… actually exactly 1, though. A tree has exactly one root.



17) Using one of the tree transversal methods, list the order of iteration through this tree.
Which ordering did you use? Preorder Postorder Inorder (Circle one)
(6 points)



Preorder: 1 2 4 7 8 5 3 6 9 0
Postorder: 7 8 4 5 2 9 0 6 3 1
Inorder: 7 4 8 2 5 1 3 9 6 0