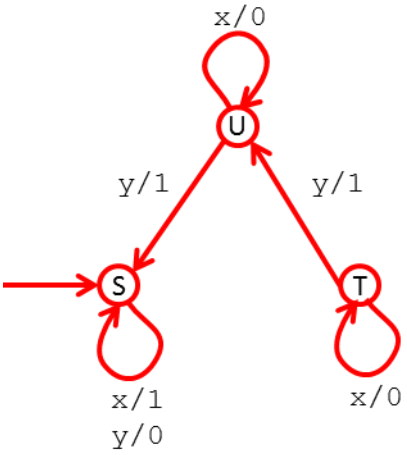
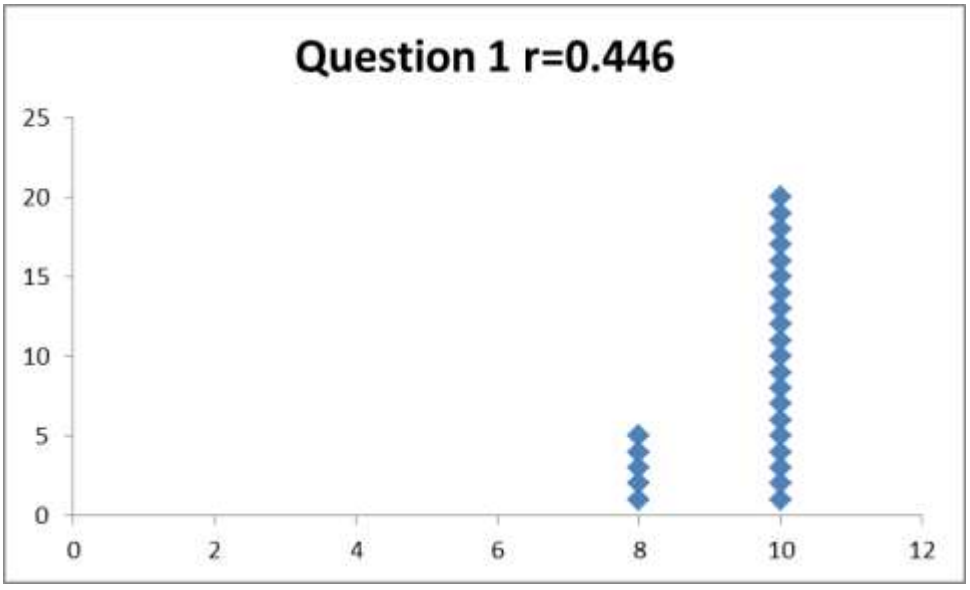
Name _____Test 3, Discrete II, Spring 2018

1) Draw the transition diagram for the finite state machine with the state transition function and output functions given below. The starting state is S.
(10 points)

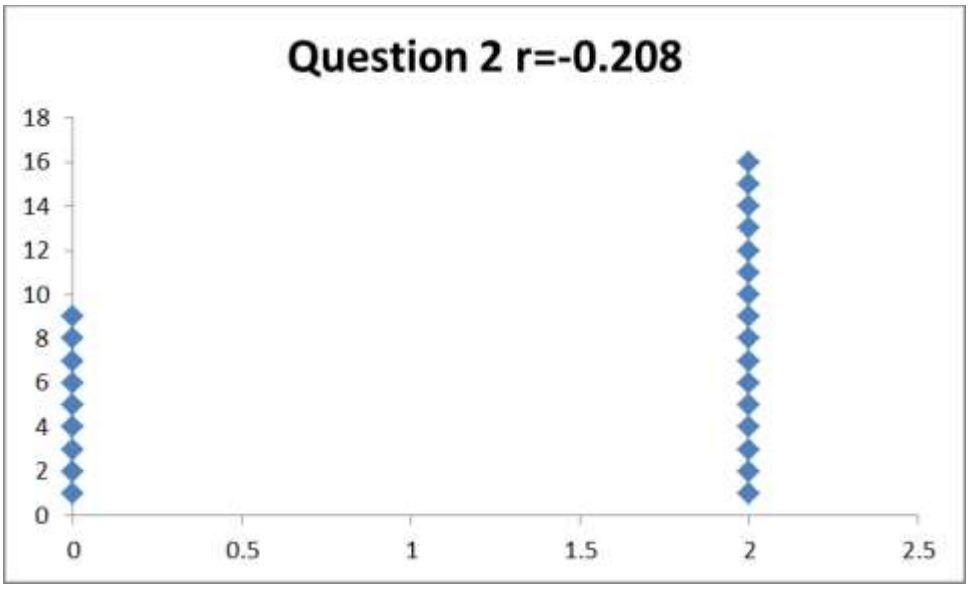
f (Next-State)		Input x	Input y
State	S	S	S
	T	T	U
	U	U	S

g (Output)		Input x	Input y
State	S	1	0
	T	0	1
	U	0	1



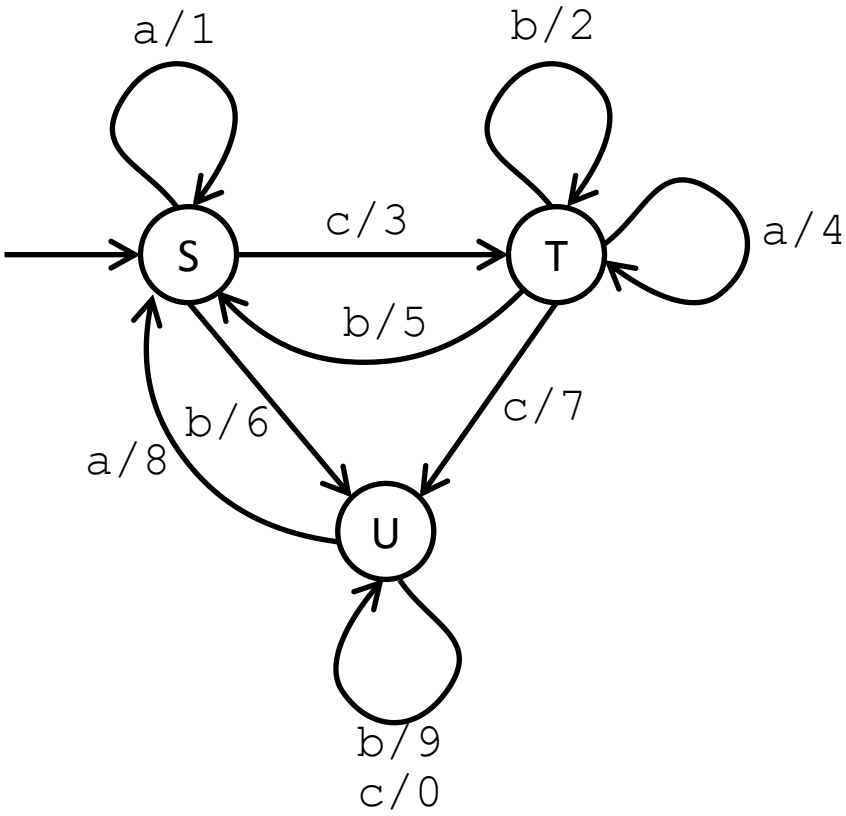


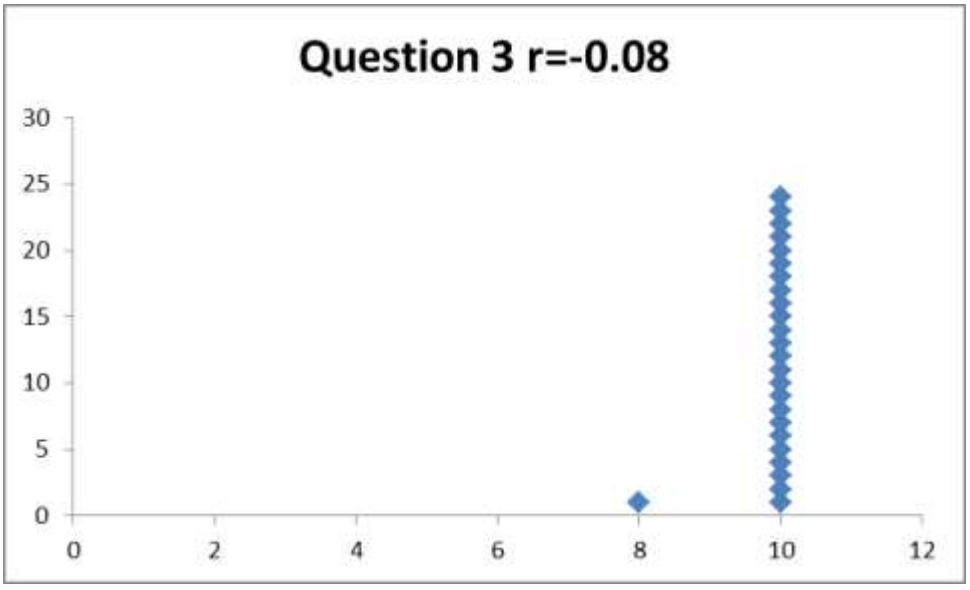
True or **False** 2) The finite state machine in the previous question is also a finite state automata.
(2 points)



3) The transition diagram for a finite state machine is below. If it is given the input $ababc$, what is the output?
(10 points)

Input: ababc
Output: 16860





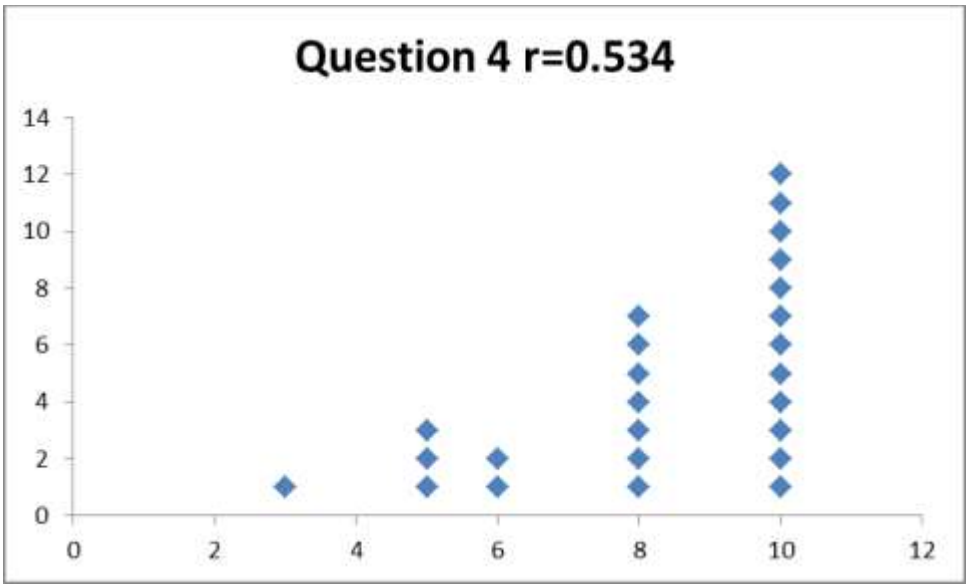
4) Given the grammar $G = (N, T, P, \sigma)$ defined below, show that $abab$ is in $L(G)$ by providing a derivation. (10 points)

$$N = \{\sigma, A, B\}$$
$$T = a, b, c$$
$$\sigma = \sigma$$

P is given by the productions below:

1. $\sigma \rightarrow AB$
2. $AB \rightarrow BA$
3. $A \rightarrow aA$
4. $B \rightarrow Bb$
5. $A \rightarrow a$
6. $B \rightarrow b$

$\sigma \rightarrow AB$ (#1)
$\Rightarrow aAB$ (#3)
$\Rightarrow aABb$ (#4)
$\Rightarrow aBAb$ (#2)
$\Rightarrow abAb$ (#5)
$\Rightarrow abab$ (#6)



5) Multiple choice – which **one** of the following **best** describes the grammar in the previous question.

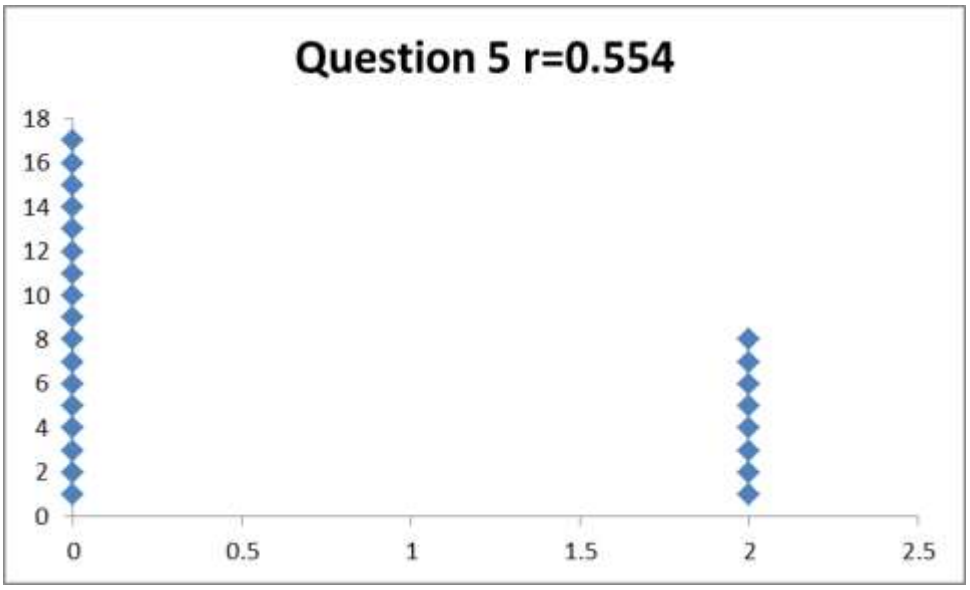
(A) General Grammar

(B) Context Sensitive Grammar

(C) Context Free Grammar

(D) Regular Grammar

(2 points)

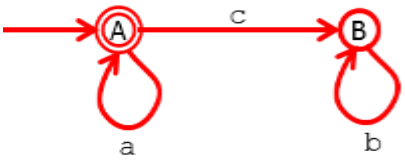


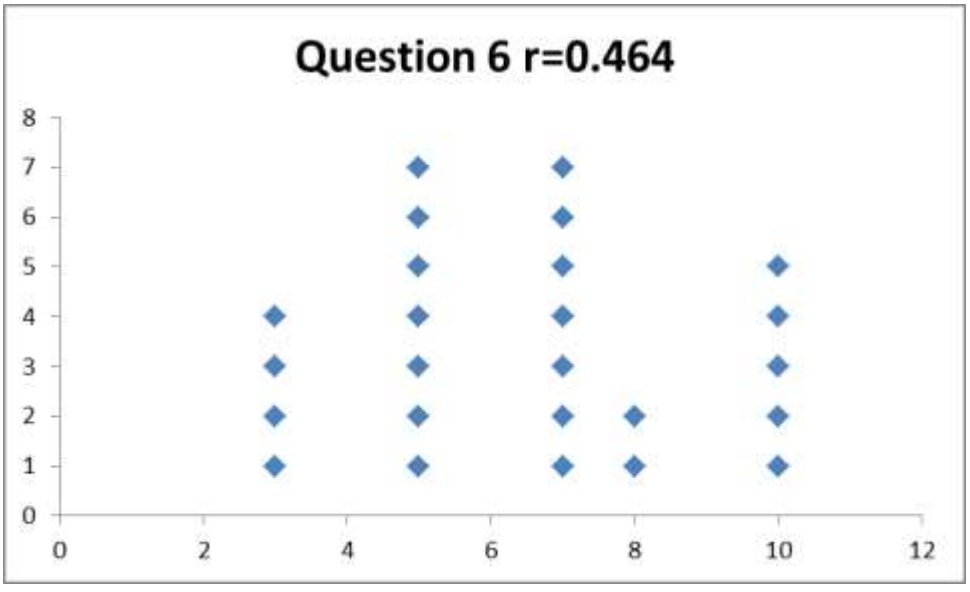
6) Given the grammar $G = (N, T, P, \sigma)$ defined below construct the transition diagram of a nondeterministic finite state automata that creates the same language. (10 points)

$$N = \{\sigma, A, B\}$$
$$T = a, b, c$$
$$\sigma = \sigma$$

P is given by the productions below:

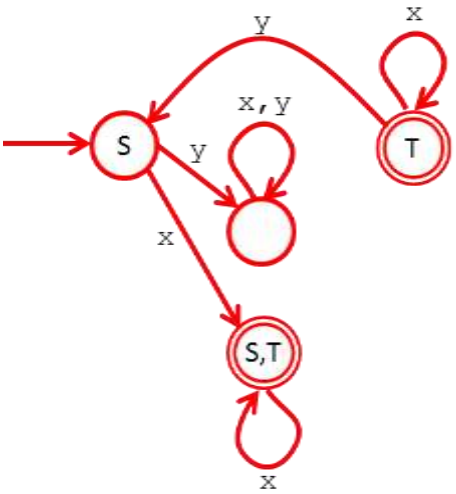
1. $\sigma \rightarrow A$
2. $A \rightarrow cB$
3. $A \rightarrow aA$
4. $B \rightarrow bB$
5. $A \rightarrow \lambda$

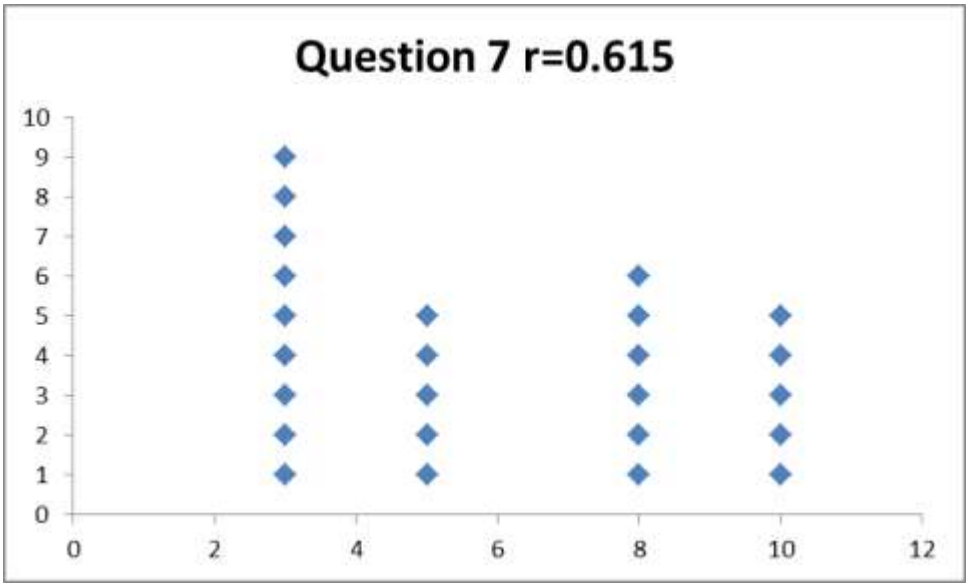




7) Given a nondeterministic finite state automata with accepting state T, starting state S, and next-state function given below, draw the transition diagram for the corresponding deterministic finite state automata. (10 points)

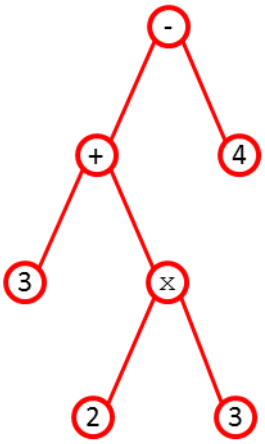
f (Next-State)		Input	
		x	y
State	S	$\{S,T\}$	$\emptyset$
	T	$\{T\}$	$\{S\}$

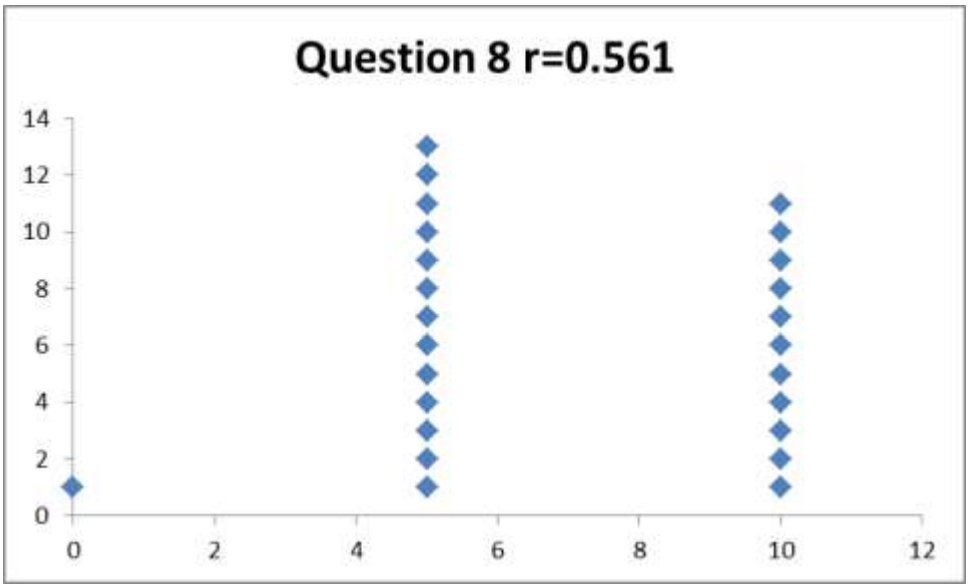




8) Given the mathematical expression below, construct the binary tree that can represent it. (10 points)

$$3 + 2 \times 3 - 4$$

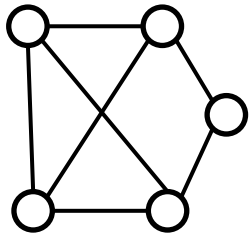


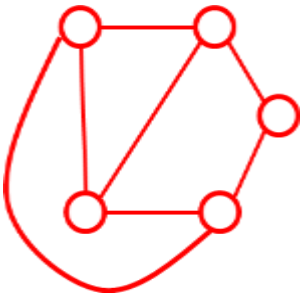


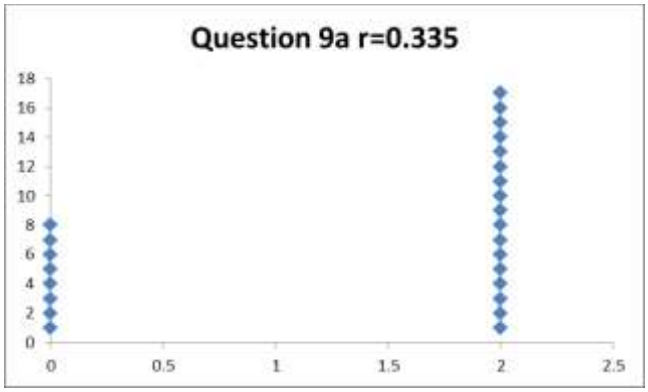
9) Is the graph below planar? Either explain why it is not, or illustrate why it is.

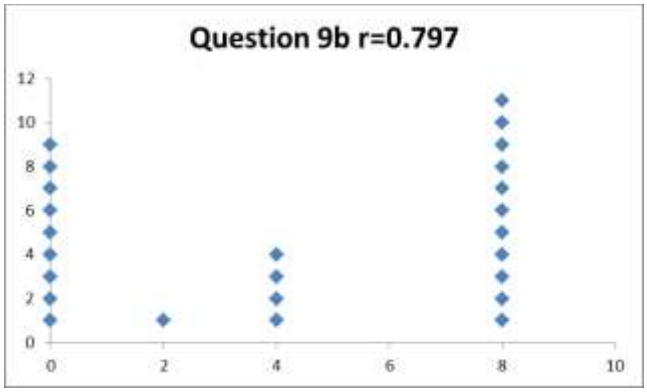
Answer: **YES** or NO
(2 points)

Justification:
(8 points)





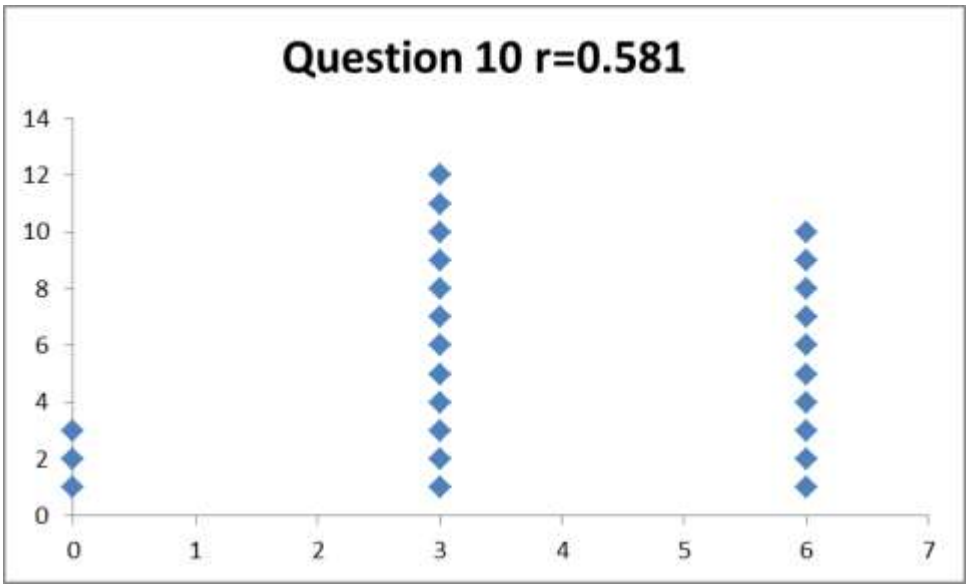




10) The expression below is given in postorder notation. Evaluate it. (6 points)

$$345 + \times$$

$345 + \times = 39 \times = 27$



11) For each of the following productions, determine to which of the following types of languages it belongs. Some problems may have multiple answers or no answer. Assume capital letters are nonterminal(N) symbols, while lowercase letters are terminal(T) symbols.
(2 points each)

(A) Context Sensitive languages
(B) Context Free languages
(C) Regular langues

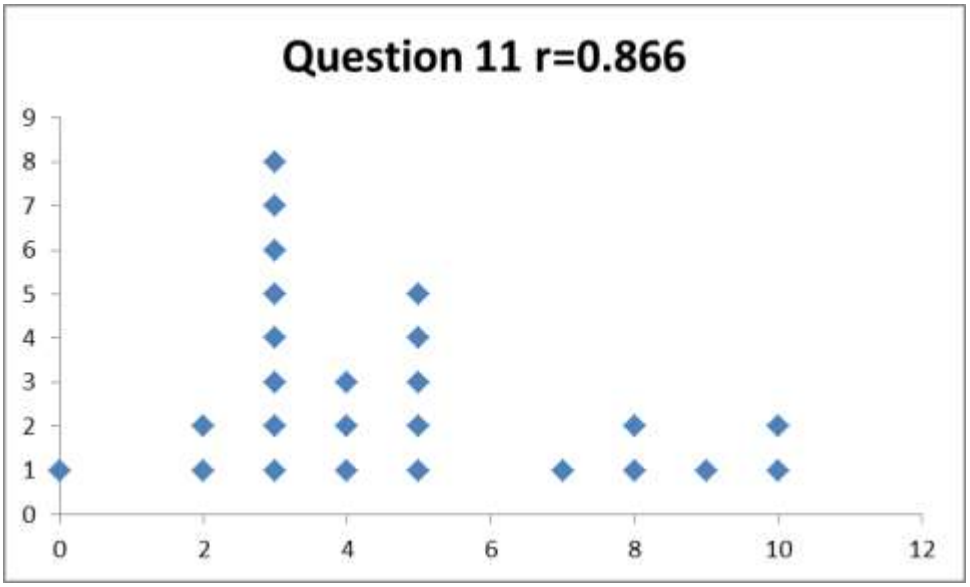
ABC_____ $A \rightarrow aA$

ABC_____ $A \rightarrow aB$

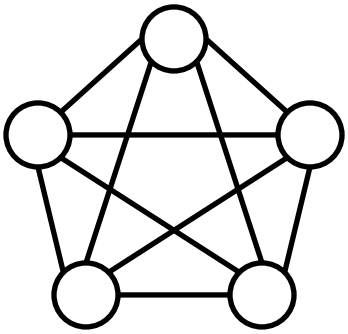
A_____ $AB \rightarrow AB$

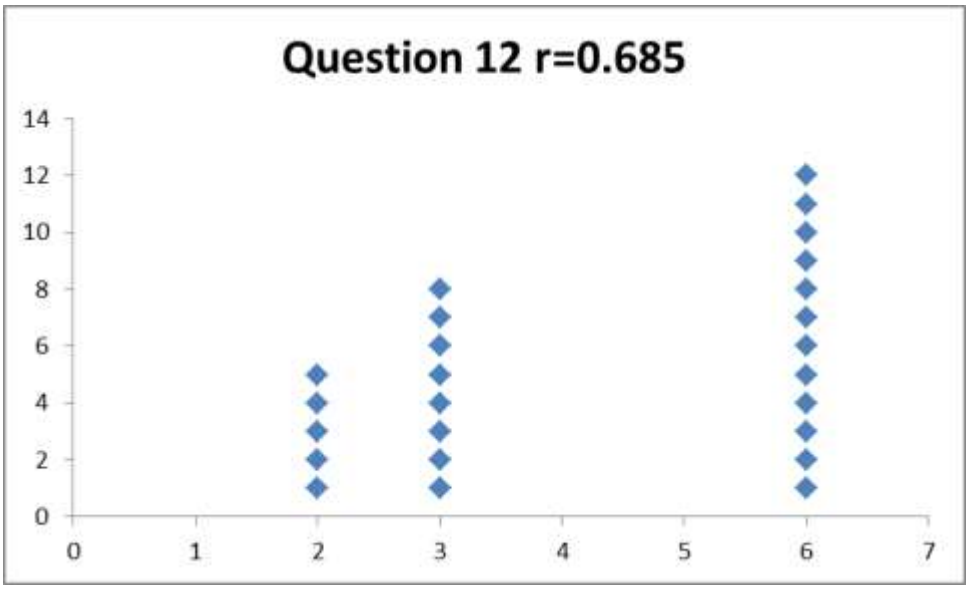
None_____ $AB \rightarrow BA$

AB_____ $A \rightarrow AabB$



12) Draw a graph that is NOT planar. (6 points)





13) What is particularly interesting about the proof of the four-color theorem? Circle one answer.
(4 points)

(A) It illustrates a clear difference between European mathematics and American mathematics.
(B) It is infeasible to prove by hand and so required the use of computers.
(C) It is an unsolved problem that can answer many questions.
(D) Its lower bound is actually larger than its upper bound.