Name ____Solutions____Quiz 1, Fields and Rings

1) Compute the following mod 23: (50 points)

$$14 + 5 \cdot 6$$

Mod 23 we have:

$$14 + 5 \cdot 6 \equiv 14 + 30 \equiv 44 \equiv 21$$

Or done another way:

$$14 + 5 \cdot 6 \equiv 14 + 30 \equiv 14 + 7 \equiv 21$$

2) Define what it means for a number in $\mathbb{Z}$ to be prime. (25 points)

A number p is prime when If $p|ab$ then $p|a$ or $p|b$ holds true for all integers a and b.

3) Define what it means for a number in $\mathbb{Z}$ to be irreducible. (25 points)

A number q is irreducible if when $q = ab$, then either $a = \pm 1$ or $b = \pm 1$ for all integers a and b.

4) Let p be a prime number in $\mathbb{Z}$. Prove that p is irreducible. (100 points)

Let p be a prime number in $\mathbb{Z}$ and write $p = ab$ for some integers a and b. Because $p = ab$, it is trivial that $p|ab$. However, p is prime so we know that $p|a$ or $p|b$. Without loss of generality let us say that $p|a$.

Next write $pk = a$ for some $k \in \mathbb{Z}$, using the fact that $p|a$. Plug that back into the original equation to obtain:

$$p = pkb$$
$$1 = kb$$

Hence $b = 1$ or $b = -1$.

5) Use the extended Euclidean Algorithm to solve $3x + 8y = 1$ for integer solutions. Show every step. (100 points)

$8 = 3 \cdot 2 + 2$ $\quad$ $2 = 8 - 3 \cdot 2$

$3 = 2 \cdot 1 + 1$ $\quad$ $1 = 3 - 2 = 3 - (8 - 3 \cdot 2) = 3 \cdot 3 - 8 = 3 \cdot 3 + 8 \cdot (-1)$

$$x = 3$$
$$y = -1$$