Name ____ Fields and Rings, Test 1, Fall 2016

1) Use the Euclidean Algorithm to find $\gcd(10,26)$. (20 points)

2) Formally define the set $\mathbb{Q}$. (20 points)

3) What is the difference between "$8 \equiv 2$ (mod 6)" and "$[8]_6 = [2]_6$" ? (20 points)

4) Solve $3x \equiv 6 \pmod{9}$ (25 points)

5) Let $f = \sum_{i=0}^{n} a_i x^i$ and $g = \sum_{j=0}^{m} b_j x^j$. Write down a formula for $f \cdot g$. (25 points)

6) Choose and formally define one of the following: (25 points)

a) $\mathbb{Q}[x]$

b) $\mathbb{Z}_n$

7) State 6 properties that a ring must satisfy. (25 points)

8) Give three different examples of rings. (20 points)

9) Explain the difference between a polynomial and a function. (20 points)

10) Let $m \in \mathbb{Z}_{\geq 2}$ and $[a]$ and $[b]$ be congruence classes mod m. Define $S \coloneqq \{x + y | x \in [a], y \in [b]\}$. Prove that $S \subseteq [a] + [b]$. (100 points)

11) Suppose $f, g \in \mathbb{Q}[x]$. Also suppose that there are infinitely many points s_i such that $f(s_i) = g(s_i)$. Prove that $f = g$. (100 points)