Codename ____ Fields and Rings, Test 2, Fall 2016

(Do not put your name on the test; write your name and codename on the code sheet)

1) Let R be a ring. Describe, in an English sentence, the set $\{x \in R | \forall_{r \in R}(xr = rx)$ (50 points)

2) Why are $x + i$ and $(1 + i)x + (-1 + i)$ associates in $\mathbb{C}[x]$? (50 points)

3) Let I and J be ideals of a ring commutative ring R. We can define an operation on these two ideals we'll call "$+$" via:

$$I + J \coloneqq \{a + b | a \in I, b \in J\}$$

Prove that $I + J$ is an ideal of R. (100 points)

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4) Let I and J be ideals of a ring commutative ring R. Refer to the previous problem for the definition of $I + J$. Prove that $I \subseteq I + J$. (50 points)

5) Let $S \subseteq \mathbb{Z}$ be the set of all prime numbers. Is S a ring? Justify your answer. (50 points)

6) Let R_1 be a ring with operations "$+$" and "$\cdot$". Also let R_2 be a ring with operations "$\oplus$" and "$\odot$".

a) Define $R_1 \times R_2$ as a set. (5 points)

b) Define the standard addition operation on $R_1 \times R_2$. (5 points)

c) Define the standard multiplication operation on $R_1 \times R_2$. (5 points)

d) Prove that $R_1 \times R_2$ has unique additive inverses under your addition operation. (35 points).

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7) In a commutative ring with unity suppose that n is the least positive integer for which we get 0 when we add 1 to itself n times; we then say R has characteristic n. If there exists no such n, we say that R has characteristic 0. For example, the characteristic of $\mathbb{Z}_5$ is 5 because $1+1+1+1+1=0$, whereas $1+1+1+1 \neq 0$.

Suppose R and S are commutative rings with unity, and there is an isomorphism $\varphi: R \to S$. Prove that if R has characteristic n, then S also has characteristic n. (100 points)