Codename ____ Fields and Rings, Test 3, Fall 2016

(Do not put your name on the test; write your name and codename on the code sheet)

This is a take-home test due Monday November 28th. We will not have class on Monday (11/21) so that you have time to work on this test.

Take-home test rules:

- You **may** use any static information such as your notes, textbook, the webpages on the internet, videos about algebra, Wikipedia, or anything you can find on Google.
- You **may not** collaborate with other people such as classmates, friends, family, posting new questions to StackExchange or the Wikipedia reference desk. This includes even seemingly innocuous information such as how difficult a classmate thinks a question is: such information can give a keen mind a clear path as to what approach to use and is thus prohibited. If a classmate asks you about the test, kindly direct them to jbeyerl@uca.edu.
- If the central ideas you present on the test are yours, you do not need to cite any sources, even if you used a source to get some information relevant to the question.
- If you manage to find the central idea to a question or a verbatim solution, you should cite the source you used.

  (Do note though that just because you find something on the internet it doesn't mean it's true. The beauty of understanding the logic in a proof is that you should know what you're writing is correct. Dr. Beyerl has found plenty of mathematical mistakes on the internet. He may even have chosen a question or two that have an easily Google-able solution that is incorrect.)

1) Prove or disprove the claim that $\mathbb{Z}_8$ and $\mathbb{Z}_4 \times \mathbb{Z}_2$ are isomorphic as rings. (50-100 points)

2) Prove or disprove the claim that $\mathbb{Z}_{10}$ and $\mathbb{Z}_5 \times \mathbb{Z}_2$ are isomorphic as rings. (50-100 points)

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3) Let R be a field and S be a ring; φ will be a homomorphism from R to S. Prove or disprove that $\ker(\varphi)$ must be $\{0_R\}$. (50-100 points)

4) Let R be a field and S be a ring; ψ will be a homomorphism from S to R. Prove or disprove that $\ker(\psi)$ must be $\{0_S\}$. (50-100 points)

5) Consider the ring $\mathbb{Z}_{17}[x]/\langle x^4+1\rangle$. Describe the ideal $\langle x^2+7\rangle$ as simply as you can. (100 points)

6) Consider the ring $\mathbb{Z}/\langle 7\rangle$. Prove that the coset $\langle 7\rangle + 20$ is equal to the coset $\langle 7\rangle + 62$. (50 points)

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7) Draw the subring tree for $\mathbb{Z}_{24}$. Identify which subrings are integral domains. (150 points)

8) In algebra, we define a monomial as the monomials you're familiar with, without the coefficient. In particular the expression "x^2y^3" is a monomial, but "$5x^2y^3$" is not a monomial. The latter is called a term. Find all monomials not in $\langle x^3y\rangle \subseteq \mathbb{Q}[x,y]$(75 points)

9) Find a proper ideal of $\mathbb{Q}[x,y]$ whose complement contains only finitely many monomials. (75 points)