Using only the definition and facts about rings below, prove the theorems below.

A ring is a set of elements with two binary operations, addition and multiplication, such that:

- Addition is closed
- Addition is commutative
- Addition is associative
- There exists an additive identity. (Do NOT call it 0 unless we have the uniqueness theorem)
- There exist additive inverses (Do NOT call them \(-a\) unless we have the uniqueness theorem)
- Multiplication is closed
- Multiplication is associative
- Multiplication distributes over addition

Theorem 1: Let \(a, b, \) and \(c\) be elements of a ring \(R\). If \(a + b = a + c\), then \(b = c\).

Theorem 2: Let \(a\) and \(b\) be elements of a ring \(R\). Then \(a + x = b\) always has a unique solution.

1) Let \(R\) be a ring. Suppose that \(a + 0_1 = a\) and \(a + 0_2 = a\) for all elements \(a \in R\). Show that \(0_1 = 0_2\).

   (This is theorem 3. You cannot use theorems 4 or 5 on this problem)

Theorem 4: For each element \(a\) in a ring \(R\), its additive inverse is unique.

Theorem 5: Let \(a\) be an element of a ring \(R\) and denote the additive identity as \(0\). Then \(a \cdot 0 = 0 \cdot a = 0\).

2) Let \(R\) be a ring and let \(a, b \in R\). Denote the additive inverse of each element \(c \in R\) as \(-c\), no matter what \(c\) is. Show that \(a(-b) = -(ab)\).