Name ____ Quiz 3

Using only the definition and facts about rings below, prove the theorems below.

A ring is a set of elements with two binary operations, addition and multiplication, such that:

- Addition is closed
- Addition is commutative
- Addition is associative
- There exists an additive identity. (Do NOT call it 0 unless we have the uniqueness theorem)
- There exist additive inverses (Do NOT call them $-a$ unless we have the uniqueness theorem)
- Multiplication is closed
- Multiplication is associative
- Multiplication distributes over addition

Theorem 1: Let a, b, and c be elements of a ring R. If $a + b = a + c$, then $b = c$.

Theorem 2: Let a and b be elements of a ring R. Then $a + x = b$ always has a unique solution.

1) Let R be a ring. Suppose that $a + 0_1 = a$ and $a + 0_2 = a$ for all elements $a \in R$. Show that $0_1 = 0_2$.
(This is theorem 3. You cannot use theorems 4 or 5 on this problem)

Theorem 4: For each element a in a ring R, it's additive inverse is unique.

Theorem 5: Let a be an element of a ring R and denote the additive identity as 0. Then $a \cdot 0 = 0 \cdot a = 0$.

2) Let R be a ring and let $a, b \in R$. Denote the additive inverse of each element $c \in R$ as $-c$, no matter what c is. Show that $a(-b) = -(ab)$.