Using only the definition and facts about rings below, prove the theorems below.

**Definition D1:** A ring is a set of elements with two binary operations, called addition and multiplication, such that:
- Addition is closed
- Addition is commutative
- Addition is associative
- There exists an additive identity. (Do NOT call it 0 unless we have the uniqueness theorem)
- There exist additive inverses (Do NOT call them $-a$ unless we have the uniqueness theorem)
- Multiplication is closed
- Multiplication is associative
- Multiplication distributes over addition

**Definition D2:** Let $R$ be a ring and $S \subseteq R$. $S$ is said to be a subring of $R$ if $S$ is itself a ring with the same operations as $R$.

1) Let $a$, $b$, and $c$ be elements of a ring $R$. Assume $a + b = a + c$, and prove that $b = c$.
   (This is theorem T1. You cannot use theorems T2+ on this problem)

**Theorem T2:** Let $a$ and $b$ be elements of a ring $R$. Then $a + x = b$ always has a unique solution.

**Theorem T3:** Let $R$ be a ring. If $a + 0_1 = a$ and $a + 0_2 = a$ for all elements $a \in R$, then $0_1 = 0_2$.

**Theorem T4:** For each element $a$ in a ring $R$, it's additive inverse is unique.

**Theorem T5:** Let $a$ be an element of a ring $R$ and denote the additive identity as 0. Then $a \cdot 0 = 0 \cdot a = 0$.

**Theorem T6:** Let $R$ be a ring and let $a, b \in R$. Denote the additive inverse of each element $c \in R$ as $-c$, no matter what $c$ is. Then $a(-b) = (-a)b = -(ab)$.

**Theorem T7:** Let $R$ be a ring, and $S$ a subset of $R$. $S$ is a subring if and only if all of the following are satisfied for all elements $a, b \in S$:
1. $S \neq \emptyset$
2. $a, b \in S \Rightarrow a + b \in S$
3. $a, b \in S \Rightarrow a \cdot b \in S$
4. $a \in S \Rightarrow -a \in S$

2) Prove that $2\mathbb{Z}$ is a ring.