1) Justify the fact that $6|18$.

(20 points)

2) Construct the division equation for $17$ divided by $5$.

(20 points)

3) An **principle ideal** $(p)$ of $\mathbb{Z}[x]$ is the set of all polynomial multiples of the polynomial $p$. Formally, that is, $(p) := \{fp | f \in \mathbb{Z}[x]\}$. Describe, in English, the principle ideal $(x - 1)$.

(40 points)
4) Use the Euclidean Algorithm to find \( \gcd(17, 5) \)
(60 points)

5) We know that \( 55 \equiv 35 \mod 10 \). Write down two equivalent statements you can derive from this.
(I mean meaningful things you can say. Don’t just say that \( 55 \equiv 35 \equiv 5 \). Use theorems that tell us interesting things.)
(40 points)
6) State Gauss’s Lemma. 
(20 points) 

7) Solve the equation $7x + 3 \equiv 5 \mod 10$. Show your work. 
(60 points)
8) We proved that the definition of addition, below, is well defined. State, precisely, what it means for this to be well defined.

\[ [a]_m + [b]_m = [a + b]_m \]

(40 points)

9) Formally define the set \( \mathbb{Q} \). Use mathematical notation.

(20 points)

10) Formally define the set \( \mathbb{Q}[x] \). Use mathematical notation.

(20 points)
11) Factor $6x^2 + 7x + 2$.

(40 points)

12) Find the product below.

\[
\left( \sum_{r=0}^{50} r^2 x^r \right) \left( \sum_{s=0}^{30} 3s x^s \right)
\]

(20 points)
13) A polynomial is called monic if the leading term is one. Prove that the product of two monic polynomials is always monic.

(100 points)
14) A polynomial in two variables can have both the variable $x$ and the variable $y$. It is said to be **homogeneous** if every term has the same degree. For example, $3x^2y - 2x^3 + xy^2$ is homogeneous of degree 3. However, $3x^2y - 2x^2 + y$ is not homogeneous.

Let $f$ and $g$ both be homogeneous polynomials. Show that their product, $f \cdot g$, is also homogeneous. (100 points)
15) Suppose $2x \equiv y \mod 100$ has a solution. Show the following two claims.

A) $2|y$
(100 points)

B) $x \equiv \frac{y}{2} \mod 50.$
(100 points)