Codename ____ Fields and Rings, Test 1, Fall 2017

(Do not put your name on the test; write your name and codename on the code sheet)

1) Justify the fact that $6|18$.

(20 points)

2) Construct the division equation for 17 divided by 5.

(20 points)

3) An principle ideal $\langle p \rangle$ of $\mathbb{Z}[x]$ is the set of all polynomial multiples of the polynomial p. Formally, that is, $\langle p \rangle \coloneqq \{fp | f \in \mathbb{Z}[x]\}$. Describe, in English, the principle ideal $\langle x - 1 \rangle$.

(40 points)

4) Use the Euclidean Algorithm to find $\gcd(17,5)$

(60 points)

5) We know that $55 \equiv 35 \bmod 10$. Write down two equivalent statements you can derive from this.

(I mean meaningful things you can say. Don't just say that $55 \equiv 35 \equiv 5$. Use theorems that tell us interesting things.)

(40 points)

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6) State Gauss's Lemma.
(20 points)

7) Solve the equation $7x + 3 \equiv 5 \bmod 10$. Show your work.
(60 points)

8) We proved that the definition of addition, below, is *well defined*. State, precisely, what it means for this to be well defined.

$$[a]_m + [b]_m = [a + b]_m$$

(40 points)

9) Formally define the set $\mathbb{Q}$. Use mathematical notation.

(20 points)

10) Formally define the set $\mathbb{Q}[x]$. Use mathematical notation.

(20 points)

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11) Factor $6x^2 + 7x + 2$.
(40 points)

12) Find the product below.

$$\left(\sum_{r=0}^{50} r^2 x^r\right)\left(\sum_{s=0}^{30} 3s\, x^s\right)$$

(20 points)

13) A polynomial is called monic if the leading term is one. Prove that the product of two monic polynomials is always monic.
(100 points)

14) A polynomial in two variables can have both the variable x and the variable y. It is said to be homogeneous if every term has the same degree. For example, $3x^2y - 2x^3 + xy^2$ is homogeneous of degree 3. However, $3x^2y - 2x^2 + y$ is not homogeneous

Let f and g both be homogeneous polynomials. Show that their product, $f \cdot g$, is also homogeneous.
(100 points)

15) Suppose $2x \equiv y \bmod 100$ has a solution. Show the following two claims.

A) $2|y$
(100 points)

B) $x \equiv \frac{y}{2} \bmod 50$.
(100 points)