1) Prove the following theorems:
- T1
- T3
- T8
- T9
- T12
- T14a
- T16a
- T17a
(100 points each)

2) Let $R$ be a commutative ring and $S$ and $T$ ideals of $R$. Define $J := \{a + b | a \in S, b \in T\}$. Prove that $J$ is an ideal of $R$.
(100 points)

3) Compute $4a$ and $a^2$ in $\mathbb{Q}[x]$ for $a = 1 + 3x^2$.

4) Compute $4a$ and $a^4$ in $\mathbb{Z}_7$ for $a = 2$.
(100 points)

5) Find all the subrings of $\mathbb{Z}_{12}$.

6) Consider $\varphi : \mathbb{Z} \to 2\mathbb{Z}$ given by $\varphi(n) = 2n$. Explain why $\varphi$ is not a ring homomorphism.
(100 points)

7) Write the number $e^{in}$ in rectangular coordinates as $a + bi$.

8) Show that in $\mathbb{Q} \times \mathbb{Z}$, the elements $(2, -1)$ and $(4, 1)$ are associates.
(100 points)

9) Explain why a field is always a PID, practically by default.

10) Give a nice description of the ideal $\langle \sqrt{7} \rangle$ in the ring $\mathbb{Z}[\sqrt{7}]$.
(100 points)

11) Factor $x^3 - 2$ into irreducibles in $\mathbb{Q}[x]$.

12) Let $\mathbb{F}$ be a field. Could the ring $\mathbb{F}[x]$ be a field? Why or why not?
(100 points)

13) Use the ring $R = \mathbb{Z}[\sqrt{2}]$ for this problem. Simplify the ideal $\langle 3 + 8\sqrt{2}, 7 \rangle$ in this ideal.
(100 points)