1) Prove the following theorems:
   - T20
   - T23
   - T24
   - T25b
   - T28a
   - T30a
   (100 points each)

2) Define $\varphi: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ via $\varphi(a, b) = a + b$. Is $\varphi$ a ring homomorphism? Justify your answer.

3) Define $\varphi: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}_6$ via $\varphi(a, b) = [a + b]_6$. Is $\varphi$ a ring homomorphism? Justify your answer.

4) Define $\varphi: \mathbb{Q}[x] \to \mathbb{Q}[x]$ via $\varphi(f) = \frac{d}{dx} f$. Is $\varphi$ a ring homomorphism? Justify your answer.
   (100 points)

5) Let $S$ be the ring of all real-valued sequences. Define $\varphi: S \to S$ as given below. Find $\ker(\varphi)$ and justify your answer.
   
   $\varphi((s_1, s_2, s_3, ...)) = (s_2, s_3, s_4, ...)$

   (100 points)

6) Let $R$ and $S$ be rings with a homomorphism $\varphi: R \to S$ between them. Find a formula for $\varphi(a^n)$ and justify your answer.
   (100 points)