Name ____ Fields and Rings, Test 3, Fall 2017

1) Prove the following theorems:

- T20
- T23
- T24
- T25b
- T28a
- T30a

(100 points each)

2) Define $\varphi: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ via $\varphi(a, b) = a + b$. Is φ a ring homomorphism? Justify your answer.
3) Define $\varphi: \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}_6$ via $\varphi(a, b) = [a + b]_6$. Is φ a ring homomorphism? Justify your answer.
4) Define $\varphi: \mathbb{Q}[x] \to \mathbb{Q}[x]$ via $\varphi(f) = \frac{d}{dx} f$. Is φ a ring homomorphism? Justify your answer.
(100 points)

5) Let S be the ring of all real-valued sequences. Define $\varphi: S \to S$ as given below. Find $\ker(\varphi)$ and justify your answer.

$$\varphi\big((s_1, s_2, s_3, \dots)\big) = (s_2, s_3, s_4, \dots)$$

(100 points)

6) Let R and S be rings with a homomorphism $\varphi: R \to S$ between them. Find a formula for $\varphi(a^n)$ and justify your answer.
(100 points)