Name ____ Test 1, Fall 2021

Part 1: Basic Knowledge

1) Let a and b be integers. What does $a|b$ mean? Be precise. (4 points)

2) Let $f, g \in \mathbb{R}[x]$. State the division equation, also known as the remainder theorem. (4 points)

3) Let $f = \sum_{k=0}^{n} a_k x^x$. What does the rational root theorem say? Be precise. (8 points)

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Part 2: Basic Skills and Concepts

4) Answer each of the following as irreducible (I), reducible (R), or not applicable (N) in $\mathbb{Z}[x]$. (8 points)

I R N a) $f = 0$

I R N b) $f = 1$

I R N c) $f = 5$

I R N d) $f = x$

I R N e) $f = x^2$

I R N f) $f = x^2 - 25$

I R N g) $f = x^2 + 25$

I R N h) $f = (x^2 - 2)(x^2 - 4)$

I R N i) $f = (x^2 + 2)(x^2 + 4)$

I R N j) $f = x^{123} - 1$

5) Let $f = 3x^2 - 2x + 1$. Give each of the following: (4 points)

a) What is the degree of f?

b) List the coefficients, separated by commas.

c) List the terms, separated by commas.

6) To which of the polynomials below does Eisenstein's theorem apply? Y=yes, N=no (4 points)

Y N a) $x^6 + 2x^4 + 4x^2 + 6$

Y N b) $x^6 + 2x^4 + 4x^2 + 8$

Y N c) $2x^6 + 4x^4 + 6x^2 + 10$

Y N d) $2x^6 + 6x^4 + 12x^2 + 24$

Y N e) $x^2 + 1$

7) Write down an expression that gives the x^{17} term of the product below. (4 points)

$$\left(\sum_{k=0}^{40} 2^k x^k\right)\left(\sum_{l=0}^{32} 3^l x^l\right)$$

8) Divide $x^3 + 1$ by $x^2 - 4$. State your answer using the division equation. (4 points)

9) Find the GCD of 15 and 70 using the Euclidean Algorithm. No credit will be given if you solve it by inspection and not the EA. (4 points)

10) Use the Extended Euclidean Algorithm to solve $15x + 70y = 5$. No credit will be given if you solve it by inspection and not the EEA. (4 points)

11) Obtain a new function by clearing all the denominators of the function below. (4 points)

$$f(x) = \frac{1}{2}x^3 + \frac{1}{3}x^2 + \frac{5}{6}x - \frac{1}{2}$$

Part 3: Proofs

12) Let $f, g \in \mathbb{Z}[x]$. Prove that $\deg(fg) = \deg(f) + \deg(g)$ (16 points)

13) Let $f \in \mathbb{Z}[x]$, and write $f = a_0 + a_1 x + \cdots + a_n x^n$. Assume If $p,$ and q are coprime integers such that $f\left(\frac{p}{q}\right) = 0$. State and prove half of the rational root theorem. (16 points)

14) Let $f, g_1, g_2, g_3 \in \mathbb{Z}[x]$ and assume f is prime. Prove that if $f | g_1 g_2 g_3$, then either $f | g_1$, $f | g_2$, or $f | g_3$. (16 points)