Name ____ Test 3, Fall 2021

Part 1: Basic Knowledge

1) Let R be a ring. What does it mean for addition to be closed?
(5 points)

2) Let R be a ring and S a subset of R. State one of the subring criterions. (5 points)

3) Let R be a ring with unity. What is the definition of 1_R that makes it so special? (5 points)

4) Let R be a ring and S a subring. What more does it take for S to be an ideal? (5 points)

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Part 2: Basic Skills and Concepts

5) Consider the ring $\mathbb{Z}_{24}$. Find $\langle 4 \rangle + 3$, what is it? (5 points)

6) Let R be an integral domain with unity. True or false: $\langle 2_R \rangle$ is always a maximal ideal, and why? (5 points)

7) Place these ideals into the same subring tree on the ring $R = \mathbb{R}[x, y]$. Note that they are not all distinct. (5 points)

a. $\langle 0 \rangle$
b. $\langle 1 \rangle$
c. $\langle 2 \rangle$
d. $\langle x \rangle$
e. $\langle x + 1 \rangle$
f. $\langle x, x + 1 \rangle$
g. $\langle y \rangle$
h. $\langle y^2 \rangle$
i. $\langle xy \rangle$

8) Let R be a ring with unity. Define $2_R \coloneqq 1_R + 1_R$. In most rings, $2_R \neq 1_R$. What is the condition required to guarantee that $2_R \neq 1_R$? (5 points)

Part 3: Proofs

Do not use any advanced theorems that would create circular logic.

9) Let R be an integral domain. State and prove what is known as the cancellation law. (20 points)

10) Let R be an integral domain with unity and let $a \in R$ be prime. Prove that a is also irreducible. (20 points)

11) Let R be a commutative ring with unity. Fix two elements $a, b \in R$. Prove that if $a = bt$ for some $t \in R$, then $\langle a \rangle \subseteq \langle b \rangle$. (20 points)